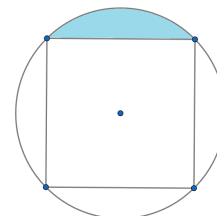


1. $\square ABCD$ is a square inscribed in a circle of diameter $7\sqrt{2}$. Find the area of the shaded region. Consider value of π as $\frac{22}{7}$.

Solution: Side of the square is 7. Area of the shaded region is one fourth of (area of circle - area of square). So, we get shaded area = $\frac{\frac{22}{7}(\frac{7\sqrt{2}}{2})^2 - 49}{4} = 7$.

Ans. 7.

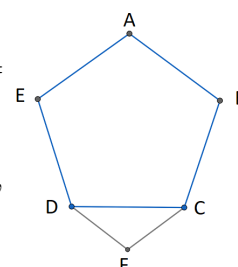


2. The distance between 2 towns is 380 km. A passenger car and a truck start moving towards each other from the 2 towns at the same time and meet each other 4 hours later. If the car drives 5 km/hr faster than the truck, what is the speed of the truck.

Solution: Since total distance travelled by them is 380 km in 4 hours, the addition of their speeds is $(380/4)=95$. So, their speeds must be 50 and 45. **Ans. 45.**

3. $ABCDE$ is a regular pentagon. $\triangle CFD$ is an isosceles triangle with $CF = DF$. $m\angle CFD = 130$. Find $m\angle ECF$.

Solution: In $\triangle CDE$, $m\angle D = 108$, and $CD = DE$, so $m\angle ECD = \frac{180-108}{2} = 36^\circ$. Similarly, $m\angle DCF = \frac{180-130}{2} = 25^\circ$. So, $m\angle ECF = 36 + 25 = 61^\circ$. **Ans. 61.**



4. Two numbers are in the ratio 2.75 : 5.25. If 13 is added to both numbers, their ratio becomes 1.8 : 3.2. Find the first number.

Solution: 2.75 : 5.25 is 11 : 21 and 1.8 : 3.2 is 9 : 16. Let the numbers be $11k$ and $21k$. So, we have $\frac{11k+13}{21k+13} = \frac{9}{16} \Rightarrow 176k + 208 = 189k + 117 \Rightarrow 13k = 91 \Rightarrow k = 7$. **Ans. 77.**

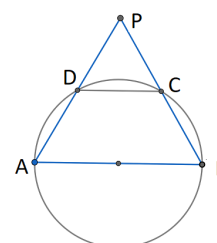
5. A tree grows by 10% of its existing height every year. If the tree is now 42.35 feet tall, how tall was it two years ago?

Solution: Let the required height be h . By compound interest formula, we get $42.35 = h(1.1)^2 = 1.21h \Rightarrow h = 35$. **Ans. 35.**

6. $\triangle PAB$ is an equilateral triangle AB is the diameter of the circle $AB = 8$. Let K be the area of $\square ABCD$. Find $2\sqrt{3} \times K$.

Solution: Let the center of the circle be O . Since $\angle A = \angle B = 60^\circ$ and $OA = OB = OC = OD = 4$, it is clear that $\triangle OAD, \triangle ODC, \triangle OBC$ are equilateral with side = 4. Area of an equilateral triangle with side a is $\frac{a^2\sqrt{3}}{4}$, $K = 3(\text{area}(\triangle OAD)) = 3(\frac{4^2\sqrt{3}}{4}) \Rightarrow 2\sqrt{3} \times K = 2\sqrt{3} \times 3(\frac{4^2\sqrt{3}}{4}) = 72$.

Ans. 72.

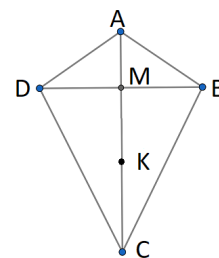


7. $\left(\frac{\sqrt{28} + \sqrt{12} + \sqrt{20}}{\sqrt{7} + \sqrt{5} + \sqrt{3}} \right)^5 =$

Solution: $\left(\frac{\sqrt{28} + \sqrt{12} + \sqrt{20}}{\sqrt{7} + \sqrt{5} + \sqrt{3}} \right)^5 = \left(\frac{2\sqrt{7} + 2\sqrt{3} + 2\sqrt{5}}{\sqrt{7} + \sqrt{5} + \sqrt{3}} \right)^5 = 2^5 = 32$. **Ans. 32.**

8. $\square ABCD$ is a kite. $AB = AD = 5$, $CB = CD = 10$, $AM = 3$. Let K be the midpoint of $\overline{MC}$. Find DK^2

Solution: $AD = 5, AM = 3 \Rightarrow DM = 4$. Using $CD = 10$, we get $CM^2 = CD^2 - DM^2 = 100 - 16 = 84 \Rightarrow MK^2 = \frac{1}{4}CM^2 = 21 \Rightarrow DK^2 = DM^2 + MK^2 = 16 + 21 = 37$. **Ans. 37.**

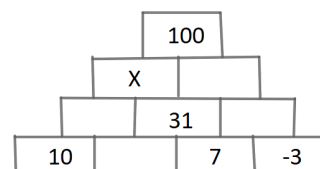


9. Four children have some toys. The first child has $\frac{1}{10}^{th}$ of the toys. The 2nd has 12 toys more than the first. The 3rd has one more toy than the first, the 4th child has double the 3rd child. How many toys are there altogether?

Solution: Suppose total toys are $10x$, so the first child has x toys. Second has $x + 12$ toys. Third has $x + 1$, so fourth has $2(x + 1)$. So, we get $x + x + 12 + x + 1 + 2(x + 1) = 10x \Rightarrow 5x = 15 \Rightarrow 10x = 30$. **Ans. 30.**

10. In the number pyramid below, the number at the top is the result of addition of the 2 numbers directly below it. Fill in the numbers in the blank squares and hence find the number in place of X .

Solution: Since 31 is the sum of 7 and the missing number in the bottom row, the missing number is 24. Once this is known, all places can be filled. So, the second row becomes 34, 31, 4. So, $X = 34 + 31 = 65$ **Ans. 65.**

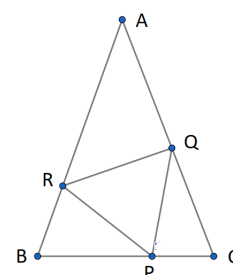


11. A Square and circle have the same perimeter. In such a case, the ratio of area of circle to the area of square is $\frac{m}{n}$ when we consider value of π as $\frac{22}{7}$, m, n are co-prime numbers. Find $m + n$.

Solution: Let the side of the square be x . So, circumference of the circle is $4x$, which means its radius is $\frac{4x}{2\pi} = \frac{(4x)(7)}{44} = \frac{7x}{11}$. So area of square = x^2 and area of circle is $\frac{22}{7} \left(\frac{7x}{11}\right)^2 = \frac{14x^2}{11}$. So, we get $\frac{m}{n} = \frac{\frac{14x^2}{11}}{x^2} = \frac{14}{11}$. **Ans. 25.**

12. In the given figure, $\triangle ABC$ is isosceles and $\triangle PQR$ is equilateral $\triangle$. $\angle ARQ = 47^\circ$ and $\angle PQC = 33^\circ$, find $\angle RPB$.

Solution: $m\angle ARQ + m\angle RAQ = m\angle RQC = m\angle RQP + m\angle PQC$
 $\Rightarrow 47 + m\angle RAQ = 60 + 33 \Rightarrow m\angle RAQ = 46$. Since $\triangle ABC$ is isosceles, we have $m\angle ABC = m\angle ACB$, so we get $m\angle ABC = \frac{180 - m\angle BAC}{2} = 67^\circ$
 Now, in $\triangle BRP$, $m\angle B + m\angle RPB = m\angle PRQ + m\angle ARQ \Rightarrow 67 + m\angle RPB = 60 + 47 \Rightarrow m\angle RPB = 40^\circ$. **Ans. 40.**

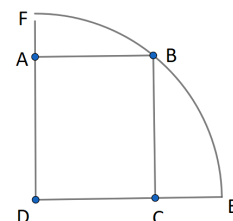


13. The product of a certain two digit number (say ab) and its digits in the reverse order (ba) is 1855. What is the sum of both the number ($ab + ba$)?

Solution: We have $1855 = 5 \times 7 \times 53$, so clearly the numbers are 35 and 53. **Ans. 88.**

14. As shown in the figure, Rectangle $ABCD$ is inscribed in a quarter circle. $AB = 3$, $BC = 4$. Find distance CE .

Solution: We have $AC = BD = DE = 5$, so $CE = DE - DC = DE - AB = 5 - 3 = 2$ **Ans. 2.**



15. A class average mark in a test is 70. The average of students who scored below 60 is 50. The average of students who scored 60 or more is 75. If the total number of students is 20, how many students scored below 60?

Solution: Total marks of the class = $70 \times 20 = 1400$. Suppose number of students who scored below 60 is x . So, their total marks are $50x$. So, number of students who scored 60 or more is $20 - x$. So, their total marks are $(20 - x)75$. So, finally we have $1400 = 50x + 75(20 - x) \Rightarrow x = 4$. **Ans. 4.**

16. If $10^{2y} = \frac{1}{25}$, find 10^{-y} .

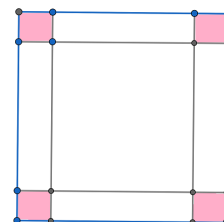
Solution: $10^{2y} = \frac{1}{25} \Rightarrow (10^{-y})^{-2} = 5^{-2} \Rightarrow 10^{-y} = 5$. **Ans. 5.**

17. If the ratio $\frac{1}{7}$ is written in decimal form, what is the digit on 2022th place after decimal point?

Solution: $\frac{1}{7} = 0.\overline{142857}$. The same six digits will be repeated. $2022 = 6 \times 337$. Which means 2022th digit will be sixth digit. **Ans. 7.**

18. A square piece of cardboard has an area of 36 cm square. A square of 1 cm $\times$ 1 cm is cut from each corner. The sides are folded in order to make an open box. What is the volume of the box in cm cubed?

Solution: Side of the original square is 6. So, after cutting the corner pieces, we have a box whose base is 4 cm $\times$ 4 cm and height 1 cm. So, volume = $4 \times 4 \times 1 = 16$. **Ans. 16.**



19. A 5 litre container full of orange juice has 2 litres of juice removed and is filled up with water and mixed thoroughly. It then has 2 litres of the mixture removed and is again filled up with water. What percentage of the final mixture is orange juice?

Solution: Original : 5 litre orange juice. 2 litre juice replaced with water. So, juice:water ratio is 3 : 2. When 2 litre of mixture is removed, we have 3 litres of mixture, which contains $\frac{9}{5}$ litre juice and $\frac{6}{5}$ litre water. So, finally we have $\frac{9}{5}$ litre juice and $\frac{6}{5} + 2 = \frac{16}{5}$ litres of water. So, ratio of juice to total is 9 : 25, i.e. juice is $\frac{9}{25} \times 100 = 36\%$. **Ans. 36.**

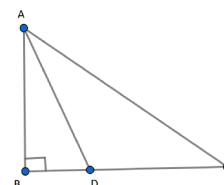
20. A computer is programmed to scan the digits of the counting numbers. For example it scans 1 2 3 4 5 6 7 8 9 10 11 12 and then it has scanned 15 digits. The computer begins its task and scans the first 1788 digits. The last counting number scanned is K ? Report sum of digits of K .

Solution: there are 9 single digit numbers, so digits scanned is 9. There are 90 two-digit numbers, so digits scanned till we reach 100 is $9 + 2(90) = 189$. There are 900 three-digit numbers, so total digits scanned till we reach 1000 is $189 + 3(900) = 2889$ which is more than 1788. So, scanning stopped after a three digit number. Number of digits scanned before reaching 100 is 189. So, number of digits scanned while scanning three digit numbers is $1788 - 189 = 1599$. So, total three-digit numbers scanned is 533. The 533rd three digit number is $533 + 99 = 632$. **Ans. 11.**

Answer Key

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans	7	45	61	77	35	72	32	37	30	65
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans	25	40	88	2	4	5	7	16	36	11

1. In $\triangle ABC$, $\angle B = 90^\circ$. $BD : DC = 1 : 2$. $AB = 20$. Area of $\triangle ADC = 140$. Find AC .



Solution: Since area of $\triangle ADC = \frac{1}{2}(AB)(DC)$, we get $140 = \frac{1}{2}(20)(DC) \Rightarrow DC = 14$. Since $BD : DC = 1 : 2$, we get $BD = 7 \Rightarrow BC = 21$. Using Pythagoras theorem in $\triangle ABC$, we get $AC^2 = AB^2 + BC^2 = 400 + 441 = 841 \Rightarrow AC = 29$. **Ans. 29.**

2. Sum of two real numbers is 28 and their product is 188. Find square of their difference.

Solution: Let the numbers be x, y . So, we have $x + y = 28$, $xy = 188$
 $\Rightarrow (x - y)^2 = (x + y)^2 - 4xy = 28^2 - 4(188) = 32$. **Ans. 32.**

3. Perimeter of a triangle is 15. How many such non congruent triangles with integer length sides are possible?

Solution: Here, we have to keep in mind that sum of two sides of a triangle is always greater than the third side. So, there cannot be a triangle of sides 1, 1, 13. So, possible triangles are (1, 7, 7), (2, 6, 7), (3, 5, 7), (3, 6, 6), (4, 4, 7), (4, 5, 6), (5, 5, 5). **Ans. 7.**

4. $(\sqrt{2} + \sqrt{3} + \sqrt{6})^2 - 4\sqrt{3} = K + 2\sqrt{3}(\sqrt{2} + \sqrt{3} + \sqrt{6})$. Find K .

Solution: $(\sqrt{2} + \sqrt{3} + \sqrt{6})^2 - 4\sqrt{3} = K + 2\sqrt{3}(\sqrt{2} + \sqrt{3} + \sqrt{6})$
 $\Rightarrow K + 4\sqrt{3} = (\sqrt{2} + \sqrt{3} + \sqrt{6})^2 - 2\sqrt{3}(\sqrt{2} + \sqrt{3} + \sqrt{6}) = (\sqrt{2} + \sqrt{3} + \sqrt{6})((\sqrt{2} + \sqrt{3} + \sqrt{6}) - 2\sqrt{3})$
 $= (\sqrt{2} + \sqrt{3} + \sqrt{6})(\sqrt{2} + \sqrt{6} - \sqrt{3})$. If we take $(\sqrt{2} + \sqrt{6}) = x$ and $\sqrt{3} = y$ then the RHS is $(x + y)(x - y) = x^2 - y^2 = (\sqrt{2} + \sqrt{6})^2 - 3 = 2 + 6 + 2(\sqrt{2})(\sqrt{6}) - 3 = 5 + 4\sqrt{3}$. So, $K = 4$. **Ans. 4.**

5. If $3^{t+1} - 3^{7-t} = 216$, find t

Solution: $3^{t+1} - 3^{7-t} = 216 \Rightarrow 3x - \frac{3^7}{x} = 216$ ($3^t = x$). So, we have $3x^2 - 216x - 2187 = 0 \Rightarrow x^2 - 72x - 729 = 0 \Rightarrow (x - 81)(x + 9) = 0$ So, $x = 81 \Rightarrow t = 4$. **Ans. 4.**

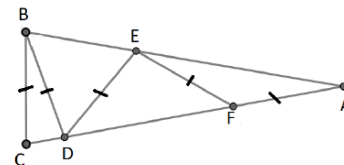
6. Arun gets 55% marks and Varun gets 67% marks and if difference in total marks they got is 84, what is the total marks of the examination?

Solution: Percentage difference in their marks is 12% and actual difference is 84, so 84 is 12% of total marks. So, total marks = 700. **Ans. 700.**

7. As shown in figure, in $\triangle ABC$, $AB = AC$, and

$BC = BD = ED = EF = AF$. Find measure of angle DEF .

Solution: Let $\angle FEA = \angle FAE = x \Rightarrow \angle EFD = \angle EDF = 2x$
 and $\angle DEF = 180 - 4x \Rightarrow \angle DEB = 180 - ((180 - 4x) + x) = 3x$.
 So, $\angle DBE = 3x \Rightarrow \angle CDB = \angle DBE + \angle A = 3x + x = 4x \Rightarrow \angle BCD = 4x$. and $\angle CBD = 180 - 8x$. But we have $\angle ACB = \angle ABC$. So, we get $4x = 180 - 8x + 3x = 180 - 5x \Rightarrow 9x = 180 \Rightarrow x = 20^\circ \Rightarrow \angle DEF = 180 - 4x = 100^\circ$. **Ans. 100.**



8. Find the GCD of 357 and 5304.

Solution: $357 = 3 \times 7 \times 17$ and $5304 = 2 \times 2 \times 2 \times 3 \times 13 \times 17$. So, $\text{GCD} = 3 \times 17 = 51$.

9. $\frac{\sqrt{343} + \sqrt{637} + \sqrt{1225}}{\sqrt{7} + \sqrt{13} + T} = 7$, find T .

Solution: $\sqrt{343} = 7\sqrt{7}$, $\sqrt{637} = 7\sqrt{13}$, $\sqrt{1225} = 7 \times 5$. So, $T = 5$. **Ans. 5.**

10. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of L is 7, find L .

Solution: $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of L means $\frac{L}{2 \times 3 \times 4 \times 5 \times 6}$, so we get $7 = \frac{L}{720} \Rightarrow L = 5040$

11. The angle between hour hand and minute hand of an analog (non digital) clock when the

time is 4 : 35 is $\left(\frac{t}{2}\right)^\circ$. Find t .

Solution: Exactly at 4 o'clock, the angle between the hands is 120° . The hour hand travels 30° in 60 minutes, so it travels $\frac{35}{2}^\circ$ in 35 minutes. The minute hand travels 360° in 60 minutes, so it advances by 210° in 35 minutes. So, angle between them at 4.35 is $210 - (120 + \frac{35}{2}) = \frac{145}{2}^\circ$. So, $t = 145$. **Ans. 145.**

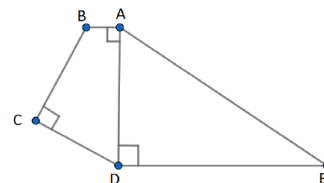
12. As shown in the figure, pentagon $ABCDE$ is such that $\angle BCD = \angle ADE = \angle DAB = 90^\circ$.

$AB = 2, BC = 7, CD = 6, DE = 12$, find AE

Solution: $BC^2 + CD^2 = BD^2 \Rightarrow BD^2 = 85$.

$AD^2 = BD^2 - AB^2 \Rightarrow AD^2 = 81$. $AE^2 = AD^2 + DE^2$

$\Rightarrow AE^2 = 81 + 144 = 225 \Rightarrow AE = 15$. **Ans. 15.**



13. Unit digit of the sum of the first 99 natural numbers is ?

Solution: Let $S = 1 + 2 + 3 + \dots + 97 + 98 + 99$. Writing it in reverse, we get

$S = 99 + 98 + 97 + \dots + 3 + 2 + 1$. If we add these two, we see that $2S = 100 + 100 + 100 + \dots$ 99 times $= 9900$. So, $S = 4950$. **Ans. 0.**

14. If $a + b + c = 0$, then $\frac{b^2 + c^2 + a^2}{b^2 - ca}$ is

Solution: $a + b + c = 0 \Rightarrow b = -(a + c) \Rightarrow b^2 = a^2 + c^2 + 2ac \Rightarrow a^2 + c^2 = b^2 - 2ac \Rightarrow \frac{b^2 + c^2 + a^2}{b^2 - ca} = \frac{2b^2 - 2ac}{b^2 - ca} = 2$. **Ans. 2.**

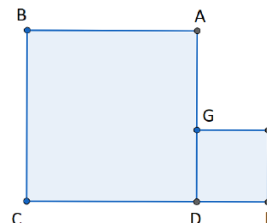
15. A circle has area 18π . Area of the largest possible square inscribed in this circle is

Solution: So, radius of the circle is $\sqrt{18} = 3\sqrt{2}$. If radius is R then the side of the square inscribed in it is $R\sqrt{2} = (3\sqrt{2})\sqrt{2} = 6$. So, answer is 6^2 . **Ans. 36.**

16. As shown in figure there are two squares. The side of each square is a whole number. The area of the figure is 58 square units, find the perimeter of the figure $ABCEFG$.

Solution: Since the sides are a whole number, very obviously, small square has side 3 and bigger square has side 7. So, Perimeter $= ED + DC + CB + BA + AG + GF + FE + ED = 3 + 7 + 7 + 7 + 4 + 3 + 3 = 34$.

Ans. 34.



17. Turning a screw driver by 90° , we can drive a screw 2 mm deeper into a piece of wood. How many complete revolutions are needed to drive a screw 8 cms into wood?

Solution: Using unitary method: for 2 mm, 90° , so for 8 cms, i.e. 80 mm, we need $\frac{80}{2} \times 90 = 3600^\circ = 10$ rotations. **Ans. 10.**

18. In a class there are boys and girls. If 15 girls leave the class the ratio of number of girls to number of boys becomes 1:2. After this if 40 boys leave, the ratio becomes 1 : 1. What is total number of students in that class in the beginning?

Solution: Let there be B boys and G girls. So, we get $G - 15 : B = 1 : 2$. Then we get $G - 15 = B - 40$. Using this in the first equation, we get $\frac{B - 40}{B} = \frac{1}{2} \Rightarrow 2B - 80 = B \Rightarrow B = 80 \Rightarrow G = 55$.

Ans. 135.

19. If $[3(230 + x)]^2 = 492a04$, then $a + x$ is

Solution: So, the number 492a04 is a perfect square which is divisible by 9. So, sum of the digits must be divisible by 9. So, we get $a = 8$. This gives $492804 = 9 \times 54756 \Rightarrow (230 + x) = 234 \Rightarrow x = 4$. **Ans. 12.**

20. The smallest positive integer to be subtracted from 2022 so that we get a perfect square is X. The smallest positive integer to be added to 2022 so that we get perfect cube is Y. Find $X + Y$.

Solution: We know $44^2 = 1936$ and $45^2 = 2025$. So, $X = 2022 - 1936 = 86$. We know $12^3 = 1728$ and $13^3 = 2197$. So, $Y = 2197 - 2022 = 175$. So, $X + Y = 261$. **Ans. 261.**

Answers

1	2	3	4	5	6	7	8	9	10
29	32	7	5	4	700	100	51	5	5040
11	12	13	14	15	16	17	18	19	20
145	15	0	2	36	34	10	135	12	261

M. Prakash Institute

Entrance Test for students going to IX std

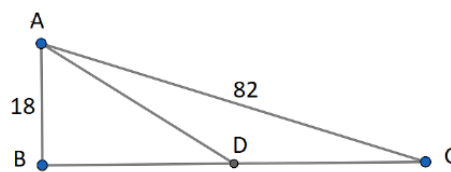
30 January 2022

7pm - 8pm

1. In $\triangle ABC$, $\angle B = 90^\circ$. D is midpoint of $\overline{BC}$. $AB = 18$, $AC = 82$. Find area of $\triangle ADC$.

Solution: Using Pythagoras theorem, we get $BC^2 = AC^2 - AB^2 = 82^2 - 18^2 = 80^2 \Rightarrow BC = 80 \Rightarrow CD = 40 \Rightarrow \text{Area of } \triangle ADC = \frac{1}{2}(AB)(CD) = \frac{1}{2}(18)(40) = 360$.

Ans. 360.



2. Sum of two natural numbers is 16 and sum of their squares is 130. Find their positive difference.

Solution: Let the numbers be x, y , so we have $x + y = 16$ and $x^2 + y^2 = 130$. But $(x + y)^2 = x^2 + y^2 + 2xy$, so we get $xy = \frac{16^2 - 130}{2} = 63$, so clearly the numbers are 7 and 9. **Ans. 2.**

3. Area of a rectangle (quadrilateral with measure of all four angles 90°) having integer sides is 144 sq. units. How many different rectangles are possible?

Solution: The possible rectangles are (1, 144), (2, 72), (3, 48), (4, 36), (6, 24), (8, 18), (9, 16), (12, 12).

Ans. 8.

4. $(2 + 3\sqrt{5})^2 = m + n\sqrt{5}$. Find $m - n$.

Solution: $(2 + 3\sqrt{5})^2 = 4 + 45 + 12\sqrt{5}$, so $m = 49$, $n = 12$. **Ans. 37.**

5. If $2^{k+1} + 2^{8-k} = 72$, find sum of all positive integer values of k .

Solution: $2^{k+1} + 2^{8-k} = 72 \Rightarrow 2(2^k) + \frac{2^8}{2^k} = 72 \Rightarrow 2x^2 - 72x + 256 = 0$ (where $x = 2^k$) $\Rightarrow x = 4$ or $x = 32 \Rightarrow k = 2, 5$. **Ans. 7.**

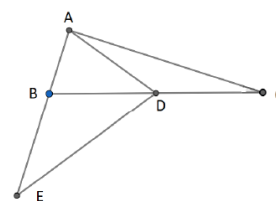
6. Arun gets 30% marks and gets 12 marks less than the passing marks. Varun gets 40% marks and gets 8 marks more than the passing marks. Find the percentage of marks required for passing.

Solution: The difference in Arun and Varun's marks is 20 and percent difference is 10%. So, Total marks (i.e. 100%) is 200. Since, passing marks are 12 more than 30% of 200 i.e. 72, its percentage is 36% **Ans. 36**

7. As shown in figure, A, B, E are collinear points. (lie on same straight line) D is point on segment BC . $BE = BD = DC = AD$. $\angle ACD = x$ and $\angle BED = 2x$. Find x .

Solution: Since $BD = DC = AD$, $\angle CAB$ is 90° . Also, $\angle DAC = x \Rightarrow \angle ADB = 2x$. Since $BD = BE$, we have $\angle BDE = \angle BED = 2x$.

Since $\angle DAC = x$, we have $\angle BAD = 90 - x$. So, in $\triangle EAD$, we have $90 - x + 2x + 2x + 2x = 180 \Rightarrow x = 18$. **Ans. 18.**



8. Find the GCD of 504, 5292.

Solution: $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$.

$5292 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7$. So, GCD = $2 \times 2 \times 3 \times 3 \times 7$. **Ans. 252.**

9. $\frac{\sqrt{338} + \sqrt{507} + \sqrt{845}}{\sqrt{5} + \sqrt{2} + \sqrt{3}} =$.

Solution: $\frac{\sqrt{338} + \sqrt{507} + \sqrt{845}}{\sqrt{5} + \sqrt{2} + \sqrt{3}} = \frac{13\sqrt{2} + 13\sqrt{3} + 13\sqrt{5}}{\sqrt{5} + \sqrt{2} + \sqrt{3}} = 13$. **Ans. 13.**

10. A basket of oranges is emptied by one person taking half of them and one more, a second person taking half of the remainder and 2 more and a third person taking half of the remainder and 3 more. How many oranges were there in the basket at first?

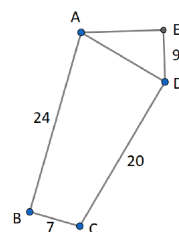
Solution: Suppose the basket has $4x$ oranges. So, the first person takes $2x + 1$. Remainder is $2x - 1$. So, Second person takes $x - \frac{1}{2} + 2 = x + \frac{3}{2}$. So, remainder is $x - \frac{5}{2}$. All of these are taken by the third person, but this is half of the remainder plus 3. So, we get $x - \frac{5}{2} = \frac{x}{2} - \frac{5}{4} + 3 \Rightarrow x = \frac{17}{2} \Rightarrow 4x = 34$. **Ans. 34.**

11. The angle between hour hand and minute hand of an analogous (non digital) clock when the time is $7 : 25$ is $(\frac{t}{2})^\circ$. Find t .

Solution: At 7, The hour hand has travelled 210° from position of 12. The minute hand is at 12. In 25 minutes, minute hand travels 150° whereas hour hand further travels by $\frac{25}{2}^\circ$. So, angle between them $= 210 + \frac{25}{2} - 150 = \frac{145}{2}^\circ$. **Ans. 145.**

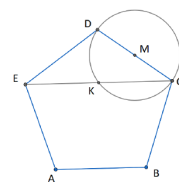
12. As shown in the figure, pentagon $ABCDE$ is such that $\angle ABC = \angle ADC = \angle AED = 90^\circ$. $AB = 24, BC = 7, CD = 20, DE = 9$, find AE

Solution: $AC^2 = AB^2 + BC^2 = 25^2$. $AD^2 = AC^2 - CD^2 = 625 - 400 = 225$ $AE^2 = AD^2 - DE^2 = 225 - 81 = 144 \Rightarrow AE = 12$. **Ans. 12.**



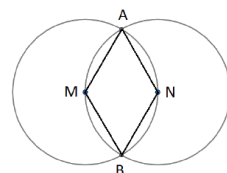
13. $ABCDE$ is a regular pentagon. Circle with $\overline{DC}$ as diameter intersects $\overline{EC}$ at K . Let M be the midpoint of $\overline{DC}$. Find $m\angle KMD$.

Solution: Every interior angle is 108° . So, in $\triangle EDC$, $\angle DEC = \angle DCE = 36^\circ$. $KM = CM$ (radius). So, $\angle MKC = 36^\circ$. So, $\angle KMD = 72^\circ$.



14. Two congruent circles with radius 2 and centers M, N intersect each other at A, B as shown in fig. Also each circle pass through center of the other. Let K be the area of $\square AMBN$. Find $\sqrt{27}K$

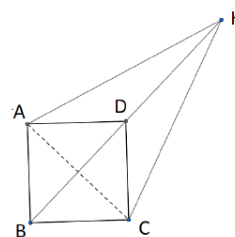
Solution: As we can see from the description, $\triangle AMN$ and $\triangle BMN$ are equilateral. Area of an equilateral triangle is $\frac{a^2\sqrt{3}}{4}$. So, here the area is $2(\frac{4\sqrt{3}}{4}) = 2\sqrt{3}$. So, $\sqrt{27}K = \sqrt{27}(2\sqrt{3}) = 18$. **Ans. 18.**



15. $\square ABCD$ is a square. Diagonal $\overline{BD}$ is extended such that $BD = DK$. Let $AB = 2$. Find $A(\triangle AKC)$.

Solution: Let the diagonals intersect at P . We have $BD = DK = 2\sqrt{2}$ and $AP = BP = CP = DP = \sqrt{2}$. So, $\text{area}(\triangle AKC) = \frac{1}{2}(AC)(PK) = 6$.

Ans. 6.



16. Given $\frac{a}{b} = \frac{3}{7}$. Find $20 \times \left(\frac{a^2+b^2}{a^2-b^2} \right)$

Solution: $20 \times \left(\frac{a^2+b^2}{a^2-b^2} \right) = 20 \times \left(\frac{\frac{a^2}{b^2} + 1}{\frac{a^2}{b^2} - 1} \right) = 20 \times \frac{\frac{9}{49} + 1}{\frac{9}{49} - 1} = -29$ **Ans. -29.**

17. $2022^2 - (2021^2 - (2020^2 - \dots - (3^2 - (2^2 - 1^1)) \dots) =$

Solution: The given sum can be written as $2022^2 - 2021^2 + 2020^2 - 2019^2 + \dots + 2^2 - 1^2$
 $= (2022 - 2021)(2022 + 2021) + (2020 - 2019)(2020 + 2019) + \dots + (2 + 1)(2 - 1)$
 $= 2022 + 2021 + 2020 + \dots + 1 = 2045253$. (because $1 + 2 + \dots + n = \frac{1}{2}n(n+1)$). **Ans. 2045253.**

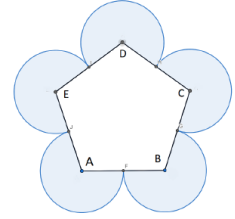
18. Find the number of integers between x and y , where $x^3 = 2022$ and $y^2 = 2022$.

Solution: $12^3 = 1728$ and $13^3 = 2197$. So, $12 < x < 13$. $24^2 = 1936$, $45^2 = 2025$. So, $44 < y < 45$. So the answer is 32. **Ans. 32.**

19. As shown in figure $ABCDE$ is regular pentagon with side $\sqrt{\frac{200}{7\pi}}$. Circles with half the side as radius and vertex as center are drawn. Find the area of shaded region.

Solution: As interior angle of the pentagon is 108° , the pentagon hides $\frac{108}{360} = \frac{3}{10}$ part of every circle. So, total area hidden by the pentagon $= \frac{3}{10} \times 5 = 1.5$ times area of a single circle. So, total shaded area is five circles minus one and half circles, i.e. 3.5 circles. Radius of the circle is half the side of the pentagon. So, radius $= \frac{1}{2} \sqrt{\frac{200}{7\pi}}$. So, area $= \frac{7}{2} \left(\pi \left(\frac{1}{2} \sqrt{\frac{200}{7\pi}} \right)^2 \right) = 25$.

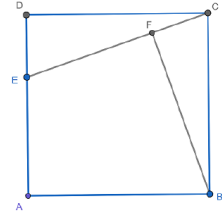
Ans. 25.



20. As shown in figure, $ABCD$ is square with side length 12. Given $DE = 5$ find distance, d of B from $\overline{CE}$. Report $13d$

Solution: $DE = 5 \Rightarrow CE = 13$.

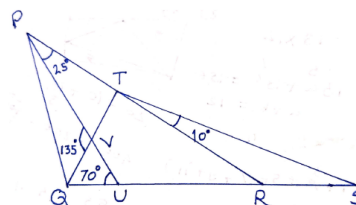
$\text{Area}(\triangle BEC) = \text{area}(\square ABCD) - \text{area}(\triangle CDE) - \text{area}(\triangle ABE)$
 $= 144 - \frac{1}{2}(5)(12) - \frac{1}{2}(7)(12) = 72$. But $\text{area}(\triangle BEC) = \frac{1}{2}(CE)(BF)$. This gives $13d = 144$. **Ans. 144.**



Answers

1	2	3	4	5	6	7	8	9	10
360	2	8	37	7	36	18	252	13	34
11	12	13	14	15	16	17	18	19	20
145	12	108	18	6	-29	2045253	32	25	144

1 In $\triangle PQR$, point U is on side QR such that $\angle PUQ = 70^\circ$ and $\angle UPR = 25^\circ$. Point T is on segment PR such that segment QT intersects segment PU at V and $\angle PVQ = 135^\circ$. Point S is on side QR extended such that $\angle STR = 10^\circ$. Find $m\angle RST$.



Solution: Since $m\angle PRU + m\angle RPU = m\angle PUQ$, we get $m\angle PRU = 45^\circ$.

Since $m\angle RST + m\angle RTS = m\angle PRU$, we get

$m\angle RST = 45 - 10 = 35^\circ$. **Ans. 35.**

2 What is the smallest positive integer that has exactly 10 factors? (for example, the number 6 has 4 factors namely 1, 2, 3, 6.)

Solution: The problem has to be solved by trying numbers. 48 is the first number satisfying the requirement. (Its factors are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48) **Ans. 48.**

3 If $\overleftarrow{n}$ denotes the number obtained by digit reversal of natural number n (For example, $\overleftarrow{583} = 385$.) then find the value of $(10 + 11 + 12 + 13 + \dots + 99) - (\overleftarrow{10} + \overleftarrow{11} + \overleftarrow{12} + \overleftarrow{13} + \dots + \overleftarrow{99})$

Solution: Observe that if a two digit number is not ending with a zero, its reversal will be a two digit number. So, when we subtract the reversals of two digit numbers, there will be exact pairing like 23, 32, 49, 94, etc. The only different numbers will be 10, 20, ..., 90 and their reversals. So, the answer is $(10 + 20 + 30 + \dots + 90) - (1 + 2 + 3 + \dots + 9) = 450 - 45 = 405$.

Ans. 405.

4 Distinct integers a, b, c, d are chosen among the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. What is the largest integer $\frac{a+b}{c+d}$ can be?

Solution: a and b must be as large as possible. So, if we take $a = 9, b = 8$, we get the numerator as prime number. So, try $a = 9, b = 7$. So, denominator cannot be 1 or 2, so must be 4, i.e. $c = 1, d = 3$. But we can make denominator smaller than 4, i.e. 3. So, numerator must be largest possible multiple of 3. It cannot be greater than 17. So, try 15, i.e. $a = 8, b = 7, c = 1, d = 2$.

Ans. 5.

5 Find the 1000th digit after the decimal point in the decimal representation of $\frac{4}{7}$.

Solution: We know that $\frac{4}{7} = 0.571428\overline{}$. The 6 digits will be repeated. The remainder when 1000 is divided by 6 is 4. So, 1000th digit will be the fourth digit in the sequence, i.e. 4.

Ans. 4

6 A train having length 500 meter crosses a man standing at the platform in 20 seconds. The same train crosses the platform in 90 seconds. Find the length of the platform in meter.

Solution: Since the train crosses a person in 20 seconds, it travels its length, i.e. 500m in 20 seconds. So, the speed of the train is 25 m/s. When the train crosses the platform, first its engine crosses the platform and the end of the train also crosses the platform. So, the train totally travels the length of the platform and its length also. In 90 seconds, the train travels $25 \times 90 = 2250$ meters. To get the length of the platform, we must subtract 500 from this.

Ans. 1750 meters.

7 The sum of two numbers is 156 and their HCF (GCD) is 13. How many such pairs exist?

Solution: Since the numbers are of the form $13k$, and $156 = 13 \times 12$, let's take values of k such that their sum is 12. If we try 1 and 11, we get 13, 143 which is an acceptable pair. If we try 2, 10, we get 26, 130, but their GCD is 26. This happens because 2 and 10 have a common factor. So, must try two numbers whose sum is 12 but they don't have anything common. The only other such pair is 5, 7. So, only two possible pairs namely (13, 143) and (65, 91). **Ans. 2.**

8 Find m if $7^{m+1} + 7^{m+2} = 2744$.

Solution: $7^{m+1} + 7^{m+2} = 2744 \Rightarrow 7^{m+1}(1 + 7) = 2744 \Rightarrow 7^{m+1} = 343 \Rightarrow 7^{m+1} = 7^3 \Rightarrow m = 2$.
Ans. 2.

9 Simplify: $\frac{\sqrt{3}+\sqrt{27}+\sqrt{75}+\sqrt{147}+\sqrt{243}+\sqrt{363}}{3\sqrt{3}}$

Solution: $\frac{\sqrt{3}+\sqrt{27}+\sqrt{75}+\sqrt{147}+\sqrt{243}+\sqrt{363}}{3\sqrt{3}} = \frac{\sqrt{3}(1+3+5+7+9+11)}{3\sqrt{3}} = 12$. **Ans. 12.**

10 If the numbers 2^{55} , 17^{14} , 31^{11} are arranged in ascending order, find the unit place digit of the greatest number among these.

Solution: Observe that $2^{55} < 2^{56} = (2^4)^{14} = 16^{14} < 17^{14}$. So, 17^{14} is greater of the first two. Also, $2^{55} = (2^5)^{11} = 32^{11} > 31^{11}$. so, 17^{14} is the largest. Observe: 17^2 has 9 in unit place, 17^3 has 3 in unit place, 17^4 has 1 in unit place, 17^5 has 7 in unit place. Now the pattern will repeat. So, the digits at unit place are 7, 9, 3, 1, 7, 9, 3, 1, ... So, 17^{14} will have 9 in unit place. **Ans. 9.**

11 The market price of a watch is Rs. 800/-. A shopkeeper gives two consecutive discounts and sells the watch at Rs.612/-. If the first discount is 10%, and if the second discount is $x\%$, find x .

Solution: The first discount is 80 Rs. So, price after first discount is 720 Rs. Since it is sold at 612/-, second discount is $720 - 612 = 108$ Rs. So, percentage discount is $\frac{108}{720} \times 100 = 15\%$.
Ans. 15.

12 m, n are positive integers such that $m + n = 2021$. Find the value of the expression $(-1)^m + (-1)^{m+1} + (-1)^{m+2} + (-1)^{m+3} + \dots + (-1)^{m+10} + (-1)^n + (-1)^{n+1} + (-1)^{n+2} + (-1)^{n+3} + \dots + (-1)^{n+10}$

Solution: Since $m + n = 2021$, one of m, n is odd and the other is even. So, $(-1)^m$ and $(-1)^n$ will have opposite signs. Their addition will be zero. Same will happen to $(-1)^{m+1}$ and $(-1)^{n+1}$. So, answer is zero. **Ans. 0.**

13 The prime factorization of the number $1^{100} \times 2^{100} \times 3^{100} \times 4^{100} \times \dots \times 10^{100}$ is written as $2^a \times 3^b \times 5^c \times 7^d$. Find $a + b + c + d$.

Solution: Observe: $1^{100} \times 2^{100} \times 3^{100} \times 4^{100} \times \dots \times 10^{100} = (1 \times 2 \times 3 \dots \times 10)^{100}$.

$4 = 2^2$, $6 = 2 \times 3$, $8 = 2^3$, $9 = 3^2$, $10 = 2 \times 5$. So,
 $(1 \times 2 \times 3 \dots \times 10)^{100} = (2 \times 3 \times 2^2 \times 5 \times (2 \times 3) \times 7 \times (2^3) \times 3^2 \times (2 \times 5))^{100} = (2^8 \times 3^4 \times 5^2 \times 7)^{100} = 2^{800} 3^{400} 5^{200} 7^{100}$ So, $a = 800, b = 400, c = 200, d = 100$. **Ans. 1500.**

14 A can finish a job in 8 days. B can finish the same job in 10 days. They finish the job together and get paid Rs. 4500/- for the whole work done. How much pay in rupees will A get?

Solution: Since B 's speed is $\frac{10}{8}$, i.e. 1.25 times that of A , he gets paid in the same ratio. So, total money is divide by 2.25, 1 part to A and 1.25 part to B . So, A gets $\frac{4500}{2.25} = 2000$ rupees.
Ans. 2000.

15 The number $12^{546} \times 63^{546} \times 29^{546}$ is written as $(a \times (a+1) \times (a+2))^{546}$, for a positive integer a . Find a .

Solution: Observe that $12 \times 63 = 27 \times 28$. So, $a = 27$ **Ans. 27.**

16 There are 30 students in a hostel. 10 new students joined so the total expense of a month increased by Rs. 600/-. But the average per head reduced by Rs. 10/-. What is the initial monthly expenditure of the hostel?

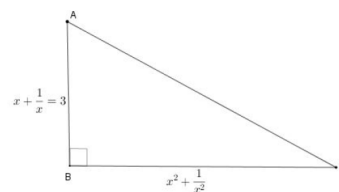
Solution: Suppose the initial average expense was x . So, total expense = $30x$.

New average = $x - 10$ and total students 40. So, total expense = $40(x - 10)$. So, we get $40(x - 10) - 30x = 600 \Rightarrow x = 100$. So, initial monthly expenditure is **Ans. 3000.**

17 In $\triangle ABC$, $m\angle ABC = 90^\circ$.

$AB = x + \frac{1}{x} = 3$, $BC = x^2 + \frac{1}{x^2}$. Find AC^2 .

Solution: $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 = 7$. So, $AC^2 = AB^2 + BC^2 = 9 + 49 = 58$. **Ans. 58.**



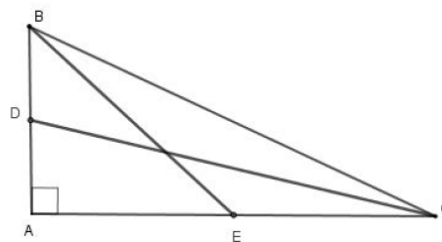
18 The ratio of milk and water in 55 litre of mixture of milk and water is 7 : 4. How many litres of water should be added to the mixture to make the ratio 7:6?

Solution: So, initially there is 35 litres of milk and 20 litres of water. Since only water is added and its ratio increases from 4 to 6, quantity added is $2 \times 5 = 10$. **Ans. 10.**

19 In $\triangle ABC$, $m\angle BAC = 90^\circ$. D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. If $CD = 7$, and $BE = 4$, then find BC^2 .

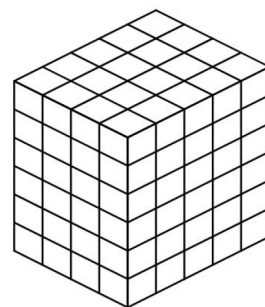
Solution: Let's denote by $a = BC$, $b = AC$, $c = AB$, the sides of the triangle. Using Pythagoras theorem in $\triangle ACD$ and $\triangle ABE$, we get $CD^2 = 49 = \frac{c^2}{4} + b^2 \Rightarrow 196 = c^2 + 4b^2$ and $BE^2 = 16 = \frac{b^2}{4} + c^2 \Rightarrow 64 = b^2 + 4c^2$. Adding these two, we get $260 = 5(b^2 + c^2) = 5(BC^2) \Rightarrow BC^2 = 52$.

Ans. 52.



20 $1 \times 1 \times 1$ cubes are glued so that a $4 \times 5 \times 6$ cuboid is formed as shown in figure. Then all the faces of the cuboid are painted and again all the cubes are separated. If m is the number of $1 \times 1 \times 1$ cubes with exactly one face painted and n is the number of $1 \times 1 \times 1$ cubes with exactly 2 faces are painted, find value of $m + n$.

Solution: A simple counting problem. $m = 2((4-2)(5-2) + (5-2)(6-2) + (6-2)(4-2)) = 52$ and $n = 4(2) + 4(3) + 4(4) = 36$. **Ans. 88.**



Answers:

1	2	3	4	5	6	7	8	9	10
35	48	405	5	4	1750	2	2	12	9
11	12	13	14	15	16	17	18	19	20
15	0	1500	2500	27	3000	58	10	52	88

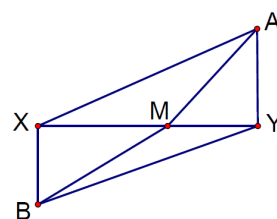
1. Virat is in the class of 16 students. In one maths exam Virat got 45 marks and the class average was 60. After receiving answer paper Virat realised that his total was wrong. He pointed it out to teacher and he got M extra marks. Now his total marks are equal to new average of the class. Find M .

Solution: When Virat's marks are 45, the total marks of the class are $60 \times 16 = 960$. With M extra marks, total of class marks is $960 + M$ and class average is $45 + M$, so we get $960 + M = 16(45 + M) \Rightarrow M = 16$. **Ans. 16.**

2. $\overline{XY}$ is line segment. M is point on $\overline{XY}$. A and B are as shown. $m\angle AXY = m\angle BXY = 90^\circ$ $AX = 11, AM = 7, BY = 10, BM = 8$. Find value of $(XM)^2 - (YM)^2$.

Solution: $AX^2 - AM^2 = (XY^2 + AY^2) - (MY^2 + AY^2) = (XY)^2 - (MY)^2$

Similarly $BY^2 - BM^2 = XY^2 - XM^2$. Subtracting second equation from the first, we get $(AX^2 - AM^2) - (BY^2 - BM^2) = XM^2 - YM^2 \Rightarrow XM^2 - YM^2 = (11^2 - 7^2) - (10^2 - 8^2) = 36$. **Ans. 36.**



3. Consider N sided regular polygons whose interior angles measured in degrees are natural numbers. What is maximum possible value of N . Report sum of digits of N .

Solution: Since interior angle is a natural number, the exterior angle is also a natural number. We want the interior angle to be the largest, so the exterior angle has to be the smallest. Since, the sum of all exterior angles (which are all equal for a regular polygon) is 360, the smallest is 1° , which means the polygon has 360 sides. **Ans. 9.**

4. The 20 digit number N is divisible by 11. Find missing digit X . $N = 60028022015X28022015$

Solution: The sum of odd place digits is $(5 + 0 + 2 + 8 + X + 1 + 2 + 0 + 2 + 0) = 20 + X$. The sum of even place digits is $(1 + 2 + 0 + 2 + 5 + 0 + 2 + 8 + 0 + 6) = 26$. So, $X = 6$. **Ans. 6.**

5. It is known that in the sequence " $A, X, B, C, D, Y, 11$ " the sum of any three consecutive terms is 19. Find the value of $A + B + C + D$

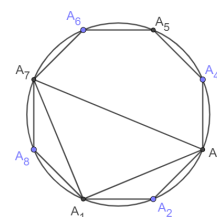
Solution: We have $11 + Y + D = 19$ and $Y + D + C = 19$ so, $C = 11$. Similarly $A = 11$. Clearly $B + C + D = 19$. So, $A + B + C + D = 30$. **Ans. 30.**

6. How many 3 digit perfect square numbers has 9 in their unit place.

Solution: If its a three digit number, which is perfect square, then we know $10^2 = 100$ and $31^2 = 961$, $32^2 = 1024$. So, the numbers whose square is the given number must be between 10 and 31. Since 9 is in unit place, the square root must have either 3 or 7 in the unit place. So, the possible numbers are 13, 17, 23, 27 (whose squares are three digit numbers with 9 in the unit place). **Ans. 4.**

7. Let $A_1, A_2, \dots, A_8$ be the 8 vertices in cyclic order of a regular octagon. (polygon with 8 sides). Find $m\angle A_1 A_3 A_7$ in degrees.

Solution: Since $A_3 A_7$ is a diameter of the circle, $m\angle A_3 A_1 A_7 = 90^\circ$. Since $\triangle A_1 A_2 A_3 \cong \triangle A_1 A_8 A_7$, we have $A_1 A_3 = A_1 A_7$, so $m\angle A_1 A_3 A_7 = 45^\circ$. **Ans. 45.**



8. A wooden $5 \times 5 \times 5$ cube is painted. Then it is cut into 125 small cubes of size $1 \times 1 \times 1$. How many of the small cubes will have at least one side painted?

Solution: Total number of cubes are 125. Out of these, if we subtract those cubes which are not painted, we get the answer. The unpainted cubes are those which are not on the surface. They form a cube which is $3 \times 3 \times 3$, i.e. 27 cubes are unpainted. So, **Ans. 98.**

9. Find the perimeter of the figure.

Solution: The only two edges (on the outer boundary) not marked are of length 6 and 10. So perimeter = $5 + 10 + 12 + 10 + 6 + 2 + 3 + 7 + 6 + 7 + 1 + 2 + 1 + 4 = 76$. **Ans. 76.**

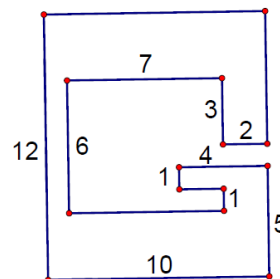


Figure for Q9

10. Side of cube is $3\sqrt{6\pi}$. What is radius of circle having area equal to surface area of cube.

Solution: Surface area of one surface of cube = $(3\sqrt{6\pi})^2 = 54\pi$. So, total surface area is 324π . If radius of the circle is r , then we get $\pi r^2 = 324\pi \Rightarrow r = 18$ **Ans. 18.**

11. $\triangle ABC$ is equilateral. Point P is in the interior of $\triangle ABC$. $\overline{PD}, \overline{PE}, \overline{PF}$ are perpendiculars from P on $\overline{BC}, \overline{CA}, \overline{AB}$ with D, E, F on corresponding sides. If $PD = 7, PF = 10$ and $PE = 8$, find distance of A from $\overline{BC}$.

Solution: Let the side of the triangle be a and let the distance of A from $\overline{BC}$ be h . We calculate the area of $\triangle ABC$ in two ways. $\text{area}(\triangle ABC) = \frac{1}{2}ah$. It is also equal to $\text{area}(\triangle PAB) + \text{area}(\triangle PBC) + \text{area}(\triangle PCA) = \frac{1}{2}a(PD + PE + PF) = \frac{1}{2}a(25)$. Equating the two, we get $h = 25$. **Ans. 25**

12. $A_1A_2 \cdots A_{15}$ is 15 sided regular polygon. A_1A_2BCDE is regular hexagon. Both are external to each other. Find measure of $\angle A_3A_2B$.

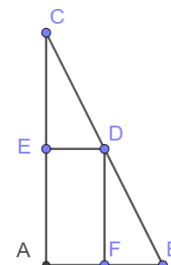
Solution: . Extend line A_1A_2 . Let P be a point on the extended line such that A_2 is between A_1 and P . Observe that $m\angle A_3A_2B = m\angle A_3A_2P + m\angle PA_2B$. The first angle is an exterior angle of a fifteen sided polygon, so its measure is $\frac{360}{15} = 24^\circ$. The second is an exterior angle of a hexagon, so it is 60° . **Ans. 84.**

13. Consider first 13 natural numbers $\{1, 2, \cdots 13\}$. One of the 13 numbers is removed. Average of remaining numbers is $\frac{27}{4}$. Which number is removed?

Solution: $1 + 2 + \cdots + 13 = 91$. Average of remaining numbers is given. So, their sum is $\frac{27}{4} \times 12 = 81$. So, number removed is $91 - 81 = 10$. **Ans. 10.**

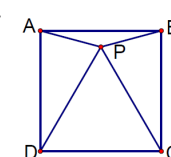
14. Distance of midpoint of ladder from ground is twice that from the wall. The height on the wall the ladder reaches is $8\sqrt{5}$ meters. Find the length of ladder in meters.

Solution: Since $DF = 2DE$, we have $AC = 2AB$, so $AB = 4\sqrt{5}$
 $\Rightarrow BC^2 = AB^2 + AC^2 = (4\sqrt{5})^2 + (8\sqrt{5})^2 = 400$. **Ans. 20.**



15. $\square ABCD$ is square. $\triangle PDC$ is equilateral triangle as shown in the figure. If $AB = 10$ find area of $\triangle APD$.

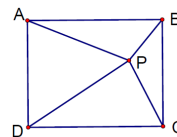
Solution: $\triangle APD \cong \triangle BPC$. Area of each triangle is half AD times $\perp$ distance of P from AD . But P is symmetrically placed. So, we get $\text{area}(\triangle APD) = \frac{1}{2}(10)5 = 25$. **Ans. 25.**



16. P is a point in the rectangle $\square ABCD$ such that $PA = 18, PB = 12, PC = 4$. Find PD .

Solution: Draw a line from P parallel to AD . Let the line intersect AB and CD in Q and R respectively. Observe that $AQ = DR$ and $BQ = CR$.

$AP^2 = AQ^2 + QP^2, BP^2 = BQ^2 + QP^2 \Rightarrow AP^2 - BP^2 = AQ^2 - BQ^2$
Similarly $DP^2 = DR^2 + RP^2, CP^2 = CR^2 + RP^2 \Rightarrow DP^2 - CP^2 = DR^2 - CR^2 =$
 $AQ^2 - BQ^2 \Rightarrow AP^2 - BP^2 = DP^2 - CP^2$ **Ans. 14.**



17. Two sides of an isosceles triangles have lengths 3 and 8 . What is the sum of all possible perimeters of triangle.

Solution: Either the sides are 3, 3, 8 or 8, 8, 3. But since sum of two sides of a triangle is always greater than the third, 3, 3, 8 is not possible. So, only possible triangle is 3, 8, 8, so perimeter is 19. **Ans. 19.**

18. There are 4 children of different integer ages under 14. The product of their ages = 450. What is the sum of their ages.

Solution: $450 = 1 \times 2 \times 3 \times 3 \times 5 \times 5$. Since all ages are under 14, we cannot have 15 as age. So, only possibility of splitting the factors is 1, 5, 9, 10. **Ans. 25.**

19. How many of the first 2020 positive integer (natural numbers) are perfect squares?

Solution: Observe that $44^2 = 1936$ and $45^2 = 2025$. So, squares of 1, 2, 3, 4, ..., 44 are perfect squares less than 2020. **Ans. 44.**

20. If 45 is sum of n consecutive positive integers what is the largest possible value of n

Solution: For largest n , we should attempt the first number as small as possible. So, try with 1. We know, $1 + 2 + \dots + 9 = 45$. So, **Ans. 9.**

21. Let x, y, z be positive integers such that $x + \frac{1}{y + \frac{1}{z}} = \frac{26}{21}$. Find value of xyz .

Solution: Observe: $\frac{26}{21} = 1 + \frac{5}{21} = 1 + \frac{1}{\frac{21}{5}} = 1 + \frac{1}{4 + \frac{1}{5}}$, so $x = 1, y = 4, z = 5$. **Ans. 20.**

22 . If $a = \sqrt{2} + 1$ what is the value of $\left(1 + \frac{1}{2 + \frac{1}{a}}\right)^2$

Solution: $\frac{1}{a} = \sqrt{2} - 1 \Rightarrow 2 + \frac{1}{a} = \sqrt{2} + 1 \Rightarrow \frac{1}{2 + \frac{1}{a}} = \sqrt{2} - 1 \Rightarrow 1 + \frac{1}{2 + \frac{1}{a}} = \sqrt{2} \Rightarrow \left(1 + \frac{1}{2 + \frac{1}{a}}\right)^2 = 2$.

Ans. 2.

23. Area of outer square is 64 . Find area of inner square.

Solution: Since area of outer square is 64, we get $AB = BC = CD = DA = 8$. It is also equal to the diameter of the circle. So, we get the radius of the circle = 4. So, $EF = EI = 4 \Rightarrow IF = 4\sqrt{2} \Rightarrow \text{Area}(\square HGIF) = (4\sqrt{2})^2 = 32$.

Ans. 32.

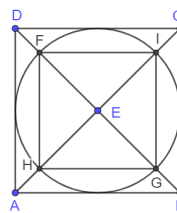


Fig for Q23

24. Sum of 5 consecutive integers is 95 . What is the largest integer?

Solution: Let the integers be $n - 4, n - 3, n - 2, n - 1, n$,
so $(n - 4) + (n - 3) + (n - 2) + (n - 1) + n = 95 \Rightarrow n = 21$. **Ans. 21.**

25. Radius of circle B is 125% of radius of circle A. Then area of circle A is $X\%$ of area of circle B. Find X .

Solution: Let the radius of circle B be r_b and radius of circle A be r_a . So, we have $\frac{r_b}{r_a} = 1.25 \Rightarrow$

$\frac{r_a}{r_b} = 0.8 \Rightarrow \frac{\pi r_a^2}{\pi r_b^2} = (0.8)^2 = 0.64$. **Ans. 64.**

26. Ten percent of the students taking a math exam fail the exam. Forty percent of the students taking the exam are boys and fifteen percent of these boys fail the exam. Twenty girls fail the exam. Find how many boys fail the exam.

Solution: The data means that boys who failed the exam are $(0.4)(0.15)$ times the total number of students, i.e. 6% is the percentage of students who fail the exam and are boys. Since, totally 10% students failed the exam, 4% is the percentage of students who fail the exam and are girls. Which is 20. percenatge of boys failing the exam is 6, which is 1.5 times that of girls. So, number of boys who fail the exam is $(20)(1.5) = 30$. **Ans. 30.**

27. How many rectangles are there in the figure?

Solution: Just count carefully ! **Ans. 36.**

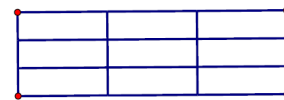


Figure for Q27

28. A takes 2 hours to wash 500 dishes and B takes 3 hours to wash 450 dishes. How long will they take working together to wash 600 dishes. (in minutes)

Solution: So, A washes $\frac{500}{120} = \frac{25}{6}$ dishes in a minute and B washes $\frac{450}{180} = \frac{5}{2}$ dishes in a minute. Together, they wash $\frac{25}{6} + \frac{5}{2} = \frac{20}{3}$ dishes in a minute, so to wash 600 dishes together, they need $(600)(\frac{3}{20}) = 90$ minutes. **Ans. 90.**

29. Number of bacteria in a bottle doubles every minute. It was found that after 15 minutes one fourth of the bottle was full. How long will it take in minutes such that the complete bottle is full of bacteria.

Solution: So, in one more minute, the number of bacteria will double, so become half the bottle. In one more minute, they will become double, so full bottle. **Ans. 17.**

30. Land is divided among the three sons Amar, Akabar and Anthony. Amar gets $\frac{2}{5}$ th of the total land. Akabar gets $\frac{1}{4}$ th of the total and Anthony gets 14 acres land. How many acres does Amar get?

Solution: Porportion of land that Anthony gets is $1 - \frac{2}{5} - \frac{1}{4} = \frac{7}{20}$ which is 14, so total land is $(14)(\frac{20}{7}) = 40$, so Amar gets $\frac{2}{5}$ th of 40, i.e. 16. **Ans. 16.**

Answers

1	2	3	4	5	6	7	8	9	10
16	36	9	6	30	4	45	98	76	18
11	12	13	14	15	16	17	18	19	20
25	84	81	20	25	14	19	25	44	9
21	22	23	24	25	26	27	28	29	30
20	2	32	21	64	30	36	90	17	16

1. Find x , if $x - 6\sqrt{x+1} + 10 = 0$.

Solution: $x - 6\sqrt{x+1} + 10 = 0 \Rightarrow x + 10 = 6\sqrt{x+1} \Rightarrow (x+10)^2 = 36(x+1)$
 $\Rightarrow x^2 - 16x + 64 = 0 \Rightarrow (x-8)^2 = 0 \Rightarrow x = 8$. **Ans. 8.**

2. 10% of N is $\frac{2}{5}$ of 33. Then $\frac{3}{4}$ of N equals.

Solution: $\frac{2}{5}$ of 33 = $33(0.4) = 13.2$. So, $N = 132 \Rightarrow \frac{3}{4}$ of N equals $\frac{3}{4}(132) = 99$. **Ans. 99.**

3. 32490 can be factorised into $a \times b$ where a, b are natural numbers and $a > b$. What is minimum difference between a and b .

Solution: $32490 = 2 \times (3^2) \times 5 \times (19)^2$. Observe that $2 \times (3^2) \times 5 = 9 \times 10$, so
 $32490 = (19 \times 9)(19 \times 10)$, so minimum difference is 19. **Ans. 19.**

4. If $a - b = 9$ and $a^2 + b^2 = 881$, where a and b are natural numbers. Find $a + b$.

Solution: $a - b = 9 \Rightarrow a^2 - 2ab + b^2 = 81$ but $a^2 + b^2 = 881$, so $2ab = 800 \Rightarrow ab = 400$. So, factorise 400 into two factors whose difference is 9, so we get $a = 25, b = 16$. **Ans. 41.**

5. Given $\frac{9^3+5^3}{9^2+5^2-x} = 14$. Find x .

Solution: $\frac{9^3+5^3}{9^2+5^2-x} = 14 \Rightarrow \frac{854}{106-x} = 14 \Rightarrow 854 = 1484 - 14x \Rightarrow 14x = 630 \Rightarrow x = 45$. **Ans. 45.**

6. L is LCM of 336, 252 and 322. G is GCD or HCF of 336, 252 and 322. Find the value of $\frac{L}{23G}$.

Solution: $336 = 2 \times 2 \times 2 \times 3 \times 7$, $252 = 2 \times 2 \times 3 \times 3 \times 7$, $322 = 2 \times 7 \times 23$. So, $GCD(336, 252, 322) = 2 \times 7$ and $LCM(336, 252, 322) = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 23$. So, $\frac{L}{23G} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 23}{23 \times 2 \times 7} = 72$. **Ans. 72.**

7. Raju's profit in first month is Rs. 1600. His profit increases by Rs. 50 per month. That is in second month his profit is Rs. 1650. In third month it is Rs. 1700 and so on. If his profit in 25th month is Rs. K then $\frac{K}{1000}$ equals.

Solution: $K = 1600 + (50)(24) = 2800$. **Ans. 2.8.**

8. Given $\sqrt[3]{7^{x+5}} = 7^5$. Find x .

Solution: $\sqrt[3]{7^{x+5}} = 7^5 \Rightarrow 7^{x+5} = (7^5)^3 = 7^{15} \Rightarrow x + 5 = 15 \Rightarrow x = 10$. **Ans. 10.**

9. Given $\frac{2\frac{2}{3}-1\frac{1}{8}}{3\frac{1}{2}+1\frac{3}{4}} = K$ then $(126K)$ equals.

Solution: $\frac{2\frac{2}{3}-1\frac{1}{8}}{3\frac{1}{2}+1\frac{3}{4}} = K \Rightarrow \frac{\frac{8}{3}-\frac{9}{8}}{\frac{7}{2}+\frac{7}{4}} = K \Rightarrow \frac{\frac{37}{24}}{\frac{21}{4}} = K \Rightarrow \frac{37}{126} = K \Rightarrow 126K = 37$. **Ans. 37.**

10. Find a natural number such that if 69 is added to $\frac{1}{8}$ th of $\frac{1}{3}$ of the number we get same natural number

Solution: The given condition is $n(\frac{1}{8})(\frac{1}{3}) + 69 = n \Rightarrow \frac{23}{24}n = 69 \Rightarrow n = 72$. **Ans. 72.**

11. If $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$ then $\frac{c^2+a^2}{c^2-a^2} = K$. Find smallest integer value of $7K$.

Solution: $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$, so $\frac{a}{c} = \frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$. So, we get $a = \frac{3}{4}c \Rightarrow a^2 = \frac{9}{16}c^2$
 $\Rightarrow \frac{c^2+a^2}{c^2-a^2} = \frac{c^2 + \frac{9}{16}c^2}{c^2 - \frac{9}{16}c^2} = \frac{25}{7} \Rightarrow 7K = 25$. **Ans. 25.**

12. Vishal completes the job in 25 days. Sachin can do the same job in 10 days. If Sachin works for 6 days then how many days Vishal will take to complete remaining job.

Solution: Vishal completes $\frac{1}{25}$ th job in one day and Sachin completes $\frac{1}{10}$ th job in one day. So, when Sachin works for 6 days, $\frac{6}{10}$ job is completed and $\frac{4}{10}$ th job is remaining. So, Vishal needs $\frac{\frac{4}{10}}{\frac{1}{25}} = 10$ days to complete remaining job. **Ans. 10.**

13. Average of Hardik in 30 cricket matches is 50 . His average in first 18 matches is 30 . What is his average in next 12 matches.

Solution: Total runs scored = $30 \times 50 = 1500$. Runs scored in first 18 matches = $18 \times 30 = 540$. So, runs scored in remaining 12 matches = $1500 - 540 = 960$, so average = $\frac{960}{12} = 80$. **Ans. 80.**

14. Average of first seven numbers in row is 45 . Average of first four is 42 . Average of last 4 is 46 . What is 4th number?

Solution: Sum of all numbers = $7 \times 45 = 315$. Sum of first four numbers is $4 \times 42 = 168$, so sum of remaining three numbers = $315 - 168 = 147$. Average of last four is 46, so their sum is $4 \times 46 = 184$. So, fourth number is $184 - 147 = 37$. **Ans. 37.**

15. How many seconds will require for a train of length 385 meter running at 63Km/Hr to cross an electric pole .

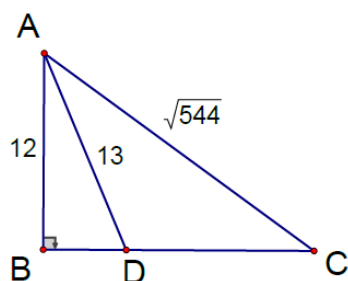
Solution: $63 \frac{\text{km}}{\text{hr}} = \frac{63000}{3600} \text{ m/sec} = \frac{35}{2} \text{ m/sec}$. So, To cross 385 meters, the train needs $385 / (\frac{35}{2}) = 22$ seconds. **Ans. 22.**

16. What is the angle between minute hand and hour hand when the clock shows time 4 hrs 10 minutes.

Solution: At 4 o'clock, the angle between the hands is 120° . In 60 minutes, the hour hand travels 30° so, in 10 minutes, it travels 5° . The minute hand travels 360° in 60 minutes, so, it travels 60° in 10 minutes. So, the angle between them is $120 + 5 - 60 = 65^\circ$. **Ans. 65.**

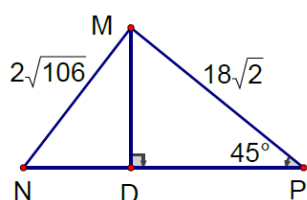
17. Measures of interior angles of triangle are $2x + 45$, $x + 10$ and $2x + 20$. Find measure of smallest angle.

Solution: Since the sum of all angles in a triangle is 180° , we get $2x + 45 + x + 10 + 2x + 20 = 180 \Rightarrow x = 21$, so the smallest angle is $x + 10 = 31^\circ$. **Ans. 31.**



18. In right angled $\triangle ABC$ as shown in figure $AB = 12$, $AD = 13$ and $AC = \sqrt{544}$. Find DC .

Solution: Using Pythagoras theorem, we get $BD = 5$, $BC = 20$, so $DC = 15$. **Ans. 15.**

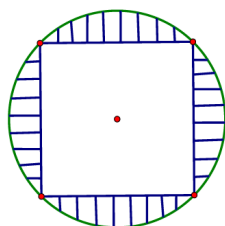


19. In $\triangle MNP$, $m\angle MPN = 45^\circ$. $\overline{MD}$ is perpendicular to $\overline{NP}$ as shown. If $MP = 18\sqrt{2}$ and $MN = 2\sqrt{106}$. Find NP .

Solution: Since $m\angle MPN = 45^\circ$, we get $MD = DP = 18$, using Pythagoras theorem in $\triangle MND$, we get $ND^2 = MN^2 - MD^2 = (2\sqrt{106})^2 - 18^2 = 100 \Rightarrow ND = 10$. **Ans. 28.**

20. Find the ratio of volume of a cube having side 12 units to surface area of the cube.

Solution: The answer is $\frac{12^3}{6(12^2)} = 2$. **Ans. 2.**



21. Square is inscribed in a circle as shown in figure. Area of shaded region is $144\pi - 288$. Find the radius of circle.

Solution: Suppose the radius is r . So, side of the square is $r\sqrt{2}$, so we get $\pi r^2 - (r\sqrt{2})^2 = 144\pi - 288 \Rightarrow r = 12$. **Ans. 12.**

22. $\square ABCD$ is trapezium. Distance between two parallel sides AB and CD is h .

$DC = 2AB = 2h$. Area of $ABCD = 384$. Find h .

Solution: Area of trapezium $= \frac{1}{2}(AB + CD)h = \frac{3h^2}{2} \Rightarrow \frac{3h^2}{2} = 384 \Rightarrow h = 16$.

Ans. 16.

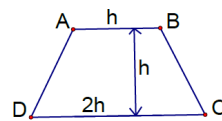
23. 6 kg of brass is made using 98.8% copper and remaining zinc. How many grams of zinc is required?

Solution: Zinc is 1.2%, so amount of zinc is $6000(\frac{1.2}{100}) = 72$. **Ans. 72.**

24. 10% of 20% of 30% of 40% of 50% of 10000 is .

Solution: 10% of 20% of 30% of 40% of 50% of 10000 $= (\frac{10}{100})(\frac{20}{100})(\frac{30}{100})(\frac{40}{100})(\frac{50}{100})10000 = 12$.

Ans. 12.



Answers

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	8	99	19	41	45	72	2.8	10	37	72	25	10
Q.No.	13	14	15	16	17	18	19	20	21	22	23	24
Ans.	80	37	22	65	31	15	28	2	12	16	72	12

1. Given $a^7 = y^2 z^2 w^2 x$, $b^7 = z^2 w^2 x^2 y$, $c^7 = w^2 x^2 y^2 z$, $d^7 = x^2 y^2 z^2 w$. If $x = \frac{b^{n_1} c^{n_2} d^{n_3}}{a^{n_4}}$ then $n_1 + n_2 + n_3 + n_4 =$

Solution: Observe that $b^7 c^7 d^7 = (z^2 w^2 x^2 y)(w^2 x^2 y^2 z)(x^2 y^2 z^2 w) = z^5 w^5 x^6 y^5$. As a trial, since the powers of y, z, w are same in the products, if we try $n_1 = n_2 = n_3$, then we get

$x = \frac{(yzw)^{\frac{5n_1}{7}} x^{\frac{6n_1}{7}}}{(y^2 z^2 w^2 x)^{\frac{n_4}{7}}}$. This gives $\frac{5n_1}{7} = \frac{2n_4}{7}$ and $\frac{6n_1}{7} - \frac{n_4}{7} = 1$. Solving these, we get $n_1 = 2, n_4 = 5$, so $n_1 + n_2 + n_3 + n_4 = 2 + 2 + 2 + 5 = 11$. **Ans. 11.**

2. In a river from point A to B , a boat takes 4 hours to travel. Same boat takes 6 hours to come back from B to A , assuming speed of boat is constant. If a wooden block is put in the same river at A , how much time in hrs. it will take to reach B .

Solution: Let the velocity of water be w and velocity of boat in still water be b . So, when the boat travels from A to B its speed is $w + b$ and when it travels from B to A , its speed is $b - w$. Let the distance between A and B be d . So, we get $\frac{d}{b+w} = 4$, $\frac{d}{b-w} = 6$. What we want is $\frac{d}{w}$. $\frac{d}{b+w} = 4 \Rightarrow \frac{b}{d} + \frac{w}{d} = \frac{1}{4}$ and $\frac{d}{b-w} = 6 \Rightarrow \frac{b}{d} - \frac{w}{d} = \frac{1}{6}$. Subtracting second equation from the first, we get $2\frac{w}{d} = \frac{1}{4} - \frac{1}{6} \Rightarrow \frac{w}{d} = \frac{1}{24} \Rightarrow \frac{d}{w} = 24$. **Ans. 24.**

3. Cost of 21 bananas and 21 oranges put together is equal to 28 apples. Instead of buying 21 bananas and 21 oranges, in the same cost if Sachin wants to buy equal number of all three fruits, find how many apples can Sachin buy?

Solution: Suppose we buy 21 bananas, 21 oranges and 28 apples. So the cost has doubled. Observe that cost of 3 bananas and 3 oranges is equal to 4 apples. So, if we buy 3 more bananas and 3 more oranges, we will have to reduce 4 apples. This will make 24 bananas, 24 oranges and 24 apples and double the cost. So, 12 bananas, 12 oranges and 12 apples is the answer. **Ans. 12.**

4. A box is filled with black and white balls. After $1/5^{\text{th}}$ of total black balls are removed ratio of number of black balls to that of white balls in the box is $2 : 3$. Then 44 white balls are removed from the box. Now the ratio of number of black balls to that of white balls now in the box is $5 : 2$. Find how many balls are now remaining in the box.

Solution: Suppose, in the end, there $5k$ black balls and $2k$ white balls. So, before removing 44 white balls, there are $5k$ black balls and $2k + 44$ white balls. Since their ratio is $2 : 3$, we get $\frac{5k}{2k+44} = \frac{2}{3} \Rightarrow 15k = 4k + 88 \Rightarrow k = 8$. So, finally there $7k$, i.e. 56 balls remaining. **Ans. 56.**

5. Find sum of all natural number n such that $n^2 + 19n + 65$ is perfect square.

Solution: Suppose $n^2 + 19n + 65 = k^2 \Rightarrow n^2 + 19n + 65 - k^2 = 0 \Rightarrow n = \frac{-19 \pm \sqrt{19^2 - 4(65 - k^2)}}{2} = \frac{-19 \pm \sqrt{4k^2 + 101}}{2}$. For n to be a natural number, we need $4k^2 + 101$ a perfect square, say, m^2 . So, $m^2 - 4k^2 = 101$. Observe that 101 is a prime number. So, we have $(m - 2k)(m + 2k) = 1 \times 101$ giving $m - 2k = 1$ and $m + 2k = 101$ which gives $m = 51, k = 25$ giving $n = \frac{-19 \pm 51}{2} = 16$ or -35 . But n is a natural number. So, it is positive. So, the only natural number possible is 16. **Ans. 16.**

6. In a rectangular garden with length 72 meters and width 30 meters, a road of fixed width is constructed as shown in the figure. Area of remaining portion of the garden is half that of original rectangular garden. Find width of the road in meters.

Solution: Let the width of the road be w . So, dimensions of inner rectangle are $72 - 2w, 30 - 2w$. So, we get $(72 - 2w)(30 - 2w) = \frac{1}{2}(72)(30) \Rightarrow w^2 - 51w + 270 = 0 \Rightarrow w = 6, 45$. **Ans. 6.**



7. A man has to reach his office at 9 : 30am. If he walks 3 km/hr, he is 10 minutes late and if he walks 4 km/hr, he is 15 minutes early. What is the distance (in km) between his house and office?

Solution: Let the distance be d km. Difference in time taken to reach office by the given different speeds is 25 minutes, i.e. $\frac{25}{60} = \frac{5}{12}$ hours. So, we have $\frac{d}{3} - \frac{d}{4} = \frac{5}{12} \Rightarrow d = 5$. **Ans. 5.**

8. What is the angle between the hour hand and the minute hand of a clock at 1:10?

Solution: At 1 o'clock, the angle between the hands is 30° . In 10 minutes, hour hand travels 5° and minute hand travels 60° . So, the angle between hand = $60 - 30 - 5 = 25^\circ$. **Ans. 25.**

9. Eight men can do a work in 12 days. After 6 days of work, 4 more men are engaged to finish the work. In how many days would remaining work be completed?

Solution: Since 8 men can do the job in 12 days, 12 men, i.e. 1.5 times the original, will need $12 \times \frac{2}{3} = 8$ days to complete the work. After 6 days, half the work is remaining. So, 12 men will need 4 days to finish this work. **Ans. 4.**

10. Let $a = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \dots$ and $b = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{6}} + \dots$, then $(\frac{a}{b} + 1)^2$ equals

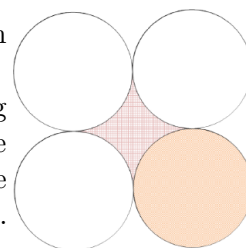
Solution: Observe that $a + b = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots$ and $b = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{6}} + \dots = \frac{1}{\sqrt{2}} \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots\right) = \frac{1}{\sqrt{2}}(a + b) \Rightarrow \frac{a}{b} + 1 = \frac{a+b}{b} = \sqrt{2}$. **Ans. 2.**

11. Three numbers a, b, c are such that $ab = 174375$ and $ac = 173600$. b is greater than c by 1. Find the sum of digits of a .

Solution: $ab - ac = 174375 - 173600 = 775$. Also, $b - c = 1 \Rightarrow ab - ac = a(b - c) = a \Rightarrow a = 775$. **Ans. 19.**

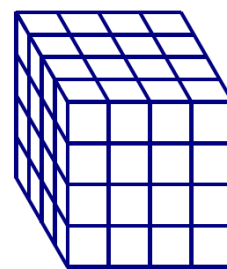
12. The diameter of all the circles is 6 cm. The circles are touching each other as shown in the figure. Find the area of shaded region.

Solution: Consider the area between the circles. Imagine a square joining all the centers of the circles. From this, at every corner, a quarter of a circle is removed, so totally, one circle area is removed. To that we are adding one circle area. So, required area is that of the square, whose side is 6. **Ans. 36.**



13. A cube of side 4 cm is painted on all of its surfaces as shown in the figure and then it is cut into smaller cubes each of side 1 cm. Let x denote the number of cubes having only two surfaces painted and y denote the number of cubes having three surfaces painted. Find $(x - y)$.

Solution: Count carefully ! $x = 24, y = 8$. **Ans. 16.**



14. If each interior angle of a regular polygon is 10 times its exterior angle then find the number of sides of the polygon.

Solution: Since sum of the interior angle and exterior angle is 180° , exterior angle is $\frac{180^\circ}{11}$. Since the sum of all exterior angles of a polygon is 360° , if the number of sides is n , we get $n \times \frac{180}{11} = 360 \Rightarrow n = 22$. **Ans. 22.**

15. The number of students in three classrooms is 138. The ratio of number of students in 1st room and 2nd room is 3 : 4 and that of 2nd room and 3rd room is 7 : 5. Find the number of students in the first room.

Solution: We can write 3 : 4 as 21 : 28 and 7 : 5 as 28 : 20. so Ratio of students in the classrooms is 21 : 28 : 20. So let the students in each classroom be $21k, 28k, 20k$. So, we get $69k = 138 \Rightarrow k = 2$. **Ans. 42.**

16. Let $k = 1 + 11 + 111 + \dots$ upto 50 terms. Find thousandth place digit in k .

Solution: Suppose you write all the numbers one below the other. There are 50 '1' in the unit

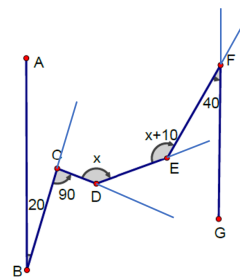
places. Which will add to 50, so unit's place is 0 and carry is 5. In tens place, there are 49 1's plus carry 5 which gives 54 so, tens place is 4 and carry 5. In hundred's place, there are 48 1's and carry 5, so hundred's place is 3 and carry 5. In thousands place, there are 47 1's and carry 5. So, thousands place is 2. **Ans. 2.**

17. How many times we should subtract GCD of 119 and 153 from LCM of 119 and 153 so that we get number equal to sum of 119 and 153.

Solution: $GCD = 17, LCM = 1071, 1071 - (119 + 153) = 799, 799/17 = 47$. **Ans. 47.**

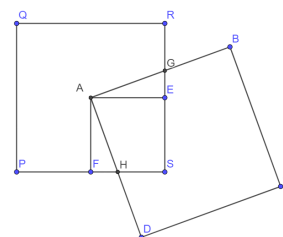
18. $\overline{AB}$ is parallel to $\overline{FG}$. Angles are as shown in figure. Find x and report answer as $x/2$.

Solution: Note that the direction of BC is 20° clockwise of north. Suppose a person walks along BC . Now he turns and moves along CD . So, he has turned 90° clockwise from direction of BC , so new direction is 110° clockwise. As he turns along DE , he has turned through $(180 - x)^\circ$ anticlockwise with respect to his direction CD . So, new direction is $110 - (180 - x) = x - 70$ clockwise with respect to north. As he turns along EF , he has turned $(180 - (x + 10)) = 170 - x$ degrees anticlockwise, so his direction is $x - 70 - (170 - x) = 2x - 240$ clockwise. But direction of line EF is 40° clockwise. So, we get $2x - 240 = 40 \Rightarrow x = 140$. **Ans. 70.**



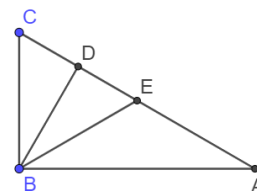
19. Two squares of side 18 each are placed in such a way that A coincides with the center of square $PQRS$ and side AB trisects side RS ($RK/KS = 1/2$). Find overlapping area of two squares.

Solution: Observe that $\triangle AEG \cong \triangle AFH$, so the overlapping area is same as $\square AFSE = 81$. **Ans. 81.**



20. In $\triangle ABC$, $\angle B = 90^\circ, \angle C = 60^\circ, \angle A = 30^\circ$. $\overline{BD} \perp \overline{AC}$, $\overline{BE}$ is angle bisector of $\angle ABD$ where D and E are on $\overline{AC}$. If $AE = 13$, find BC .

Solution: Observe that $m\angle EBA = 30^\circ$, so $BE = AE = 13$. Also, $m\angle EBC = m\angle ECB = 60^\circ$, so $\triangle EBC$ is equilateral, so $BC = BE = 13$. **Ans. 13.**



21. Tap A can fill a tank in 4 hours while tap B can empty it in 6 hours. If both taps are opened together at the same time when the tank is empty, then find the number of hours required to fill the tank completely.

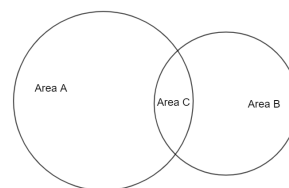
Solution: Tap A fills $\frac{1}{4}^{th}$ tank in 1 hour and tap B empties $\frac{1}{6}^{th}$ tank in one hour. So, when both taps are open, they fill $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ tank in 1 hour. So, they need 12 hours to fill the tank. **Ans. 12.**

22. Two circles with radii $\frac{7}{\sqrt{\pi}}$ and $\frac{4}{\sqrt{\pi}}$ intersect each other. Find difference in areas of non intersecting parts of these circles.

Solution: What we want to find out is

$Area A - Area B = (Area A + Area C) - (Area B + Area C)$ which is nothing but difference in areas of the circles. So, the answer is

$$\pi\left(\frac{7}{\sqrt{\pi}}\right)^2 - \pi\left(\frac{4}{\sqrt{\pi}}\right)^2 = 33.$$



23. Three natural numbers x, y, z leave remainders 18, 21, 20 respectively. When divided by 23. Find the remainder when $(x + y + z)$ is divided by 23

Solution: Suppose $x = 23n_1 + 18, y = 23n_2 + 21, z = 23n_3 + 20$, so $x + y + z = 23(n_1 + n_2 + n_3) + 18 + 21 + 20 = 23(n_1 + n_2 + n_3 + 2) + 13$. **Ans. 13.**

24. 100 rupee change is to be tendered in the denominations of Rs.20, Rs. 10 and Rs. 5 notes. The only condition is that, the tendered change must have at least one note from each

denomination. How many combination are possible? For example: One possible combination is 4 Rs.20, 1 Rs. 10 and 2 Rs. 5 note.

Solution: Since we need atleast one note of each type, that makes 35 rupees. Let's keep it aside, so it leaves 65 rupees. Now, we have to give 65 rupees by any combination of the notes. The following is the possible ways:

Sr.No.	20	10	5
1	3	0	1
2	2	2	1
3	2	1	3
4	2	0	5
5	1	4	1
6	1	3	3
7	1	2	5
8	1	1	7
9	1	0	9
10	0	6	1
11	0	5	3
12	0	4	5
13	0	3	7
14	0	2	9
15	0	1	11
16	0	0	12

Ans. 16

Answers

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	11	24	12	56	16	6	5	25	4	2	19	36
Q.No.	13	14	15	16	17	18	19	20	21	22	23	24
Ans.	16	22	42	2	47	70	81	13	12	33	13	16

1. The ratio between the length and the perimeter of a rectangular plot is $1 : 3$. The ratio between the length and the breadth of the plot is $k : 1$. Find k .

Solution: Let the length and breadth be kx and x . So, perimeter $= 2x(k + 1)$. So, we get $\frac{kx}{2x(k+1)} = \frac{1}{3} \Rightarrow 3k = 2k + 2 \Rightarrow k = 2$. **Ans. 2.**

2. If $mn = 1$, $a + b = 0$ and $m + a = 32$, then find the value of $\left[\frac{(m + \frac{1}{n})^2 - \frac{2m}{n}}{m + \frac{1}{n}} \right] + \left[\frac{a^2 + b^2}{a - b} \right]$.

Solution: Express everything in terms of m . $a = 32 - m$, $b = m - 32$, $n = \frac{1}{m}$. This gives $\left[\frac{(m + \frac{1}{n})^2 - \frac{2m}{n}}{m + \frac{1}{n}} \right] + \left[\frac{a^2 + b^2}{a - b} \right] = \left[\frac{(m + m)^2 - 2m^2}{2m} \right] + \left[\frac{(32 - m)^2 + (m - 32)^2}{32 - m - (m - 32)} \right] = 32$.

Ans. 32.

3. Find the value of $\frac{\sqrt[3]{81} + \sqrt[3]{-192} + \sqrt[3]{375}}{\sqrt[3]{24}}$.

Solution: $\frac{\sqrt[3]{81} + \sqrt[3]{-192} + \sqrt[3]{375}}{\sqrt[3]{24}} = \frac{3\sqrt[3]{3} - 4\sqrt[3]{3} + 5\sqrt[3]{3}}{2\sqrt[3]{3}} = 2$. **Ans. 2.**

4. The circumference of front wheel of a wagon is 2π meters and the circumference of back wheels are 3π meters. To cover x meters distance, front wheels have made 7 more revolutions than the back wheels. Find value of $x - 100$.

Solution: Number of revolutions to cover x meters for back wheel $= \frac{x}{3\pi}$ and for front wheel $= \frac{x}{2\pi} \Rightarrow \frac{x}{2\pi} - \frac{x}{3\pi} = 7 \Rightarrow x = (6\pi)(7)$. Using $\pi = \frac{22}{7}$, we get $x = 132$. **Ans. 32.**

5. In the given figure, bisectors of $\angle B$ and $\angle D$ of quadrilateral $ABCD$ meet ray CD and ray AB at P and Q respectively. If $\angle A + \angle C = 170^\circ$, Find $\angle BPC + \angle AQD$.

Solution: Sum of angles in a quadrilateral is 360° .

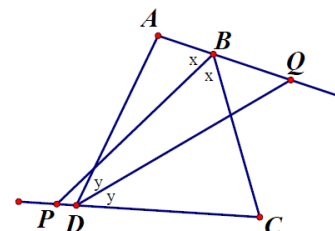
In $\square ABCD$, $\angle A + 2x + \angle C + 2y = 360 \Rightarrow x + y = 95$.

$\angle QDP = 180 - y$ and $\angle PBQ = 180 - x$

So in $\square BQDP$, $\angle PBQ + \angle BQD + \angle QDP + \angle DPB = 360$

$\Rightarrow 180 - x + \angle BQD + 180 - y + \angle DPB = 360$

$\Rightarrow \angle BQD + \angle DPB = x + y = 95$. **Ans. 95.**



6. Water is pouring into a cubical reservoir at the rate of 60 liters per minute. If the volume of reservoir is 108 m^3 , find the number of hours it will take to fill the reservoir.

Solution: Rate of filling of reservoir $= 60$ liters per minute $= 3600$ liters per hour.

$1 \text{ m}^3 = 1000$ liters. So, time required in hours to fill the reservoir $= \frac{108000}{3600} = 30$. **Ans. 30.**

7. There are two types of workers in a workshop, skilled and unskilled. The average salary of all workers in a workshop is Rs. 10,600. The average salary of 8 skilled workers is Rs. 15,000 and the average salary of all remaining unskilled workers is Rs. 9,000. Find the total number of workers in the workshop.

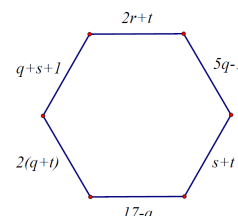
Solution: Suppose the total number of workers is n . So, number of unskilled workers $= n - 8$. So, we get $n(10600) = (8)(15000) + (n - 8)(9000) \Rightarrow n = 30$. **Ans. 30.**

8. Sides of regular hexagon are as shown. Find value of r .

Solution: We have $q + s + 1 = s + t \Rightarrow t = q + 1$

$2(q + t) = 5q - 1 \Rightarrow 2(q + q + 1) = 5q - 1 \Rightarrow q = 3 \Rightarrow t = 4$

$2r + t = 17 - q \Rightarrow 2r + 4 = 17 - 3 \Rightarrow r = 5$. **Ans. 5.**



9. The value of a TV set depreciates every year by 25%. Present value of a TV set is Rs. 45,000. If it's value after one year is $(1250 \times n)$ Find n .

Solution: $(45000)\frac{3}{4} = 1250 \times n \Rightarrow n = \frac{(45000)(3)}{(1250)(4)} = 27$ **Ans. 27.**

10. If $\frac{3^{3.5} \times 21^2 \times 42^{2.5} \times 7^{3.5}}{2^{2.5}} = 21^x$. Find the value of x .

Solution: $\frac{3^{3.5} \times 21^2 \times 42^{2.5} \times 7^{3.5}}{2^{2.5}} = 21^x \Rightarrow \frac{3^{3.5} \times 3^2 \times 7^2 \times 2^{2.5} \times 3^{2.5} \times 7^{2.5} \times 7^{3.5}}{2^{2.5}} = 21^x \Rightarrow x = 8$ **Ans. 8**

11. Length, breadth and height of the rectangular box are denotes as L, B, H respectively $L : B : H = 2 : 3 : 4$, Difference between the cost of covering it with sheet of paper at the rates Rs. 8 per m^2 and Rs. 9.50 per m^2 is Rs. 1,248. Find the height of the box.

Solution: Difference in the cost is 1.50 per m^2 and it is 1248, so area = $\frac{1248}{1.5} = 832 m^2$. Let L, B, H be $2x, 3x, 4x$. So we get $2((2x)(3x) + (3x)(4x) + (4x)(2x)) = 832 \Rightarrow x = 4$ **Ans. 16.**

12. If $\frac{2}{3}(4x - 1) - \{4x - (\frac{1-3x}{2})\} = \frac{x-7}{2}$ then find x .

Solution: Multiplying both sides by 6, we get $16x - 4 - 24x + 3 - 9x = 3x - 21 \Rightarrow x = 1$ **Ans. 1.**

13. Rohit secured 42% of the total marks and secured 10 marks more than the passing marks. In the same exam, Mohit scored 29% of the total marks and failed by 16 marks. What was the passing percentage?

Solution: Percentage difference between Rohit's and Mohit's marks is 13. Actual marks difference is 26. Which means 100 % marks means 200 marks. So, from Rohit's marks, we conclude that passing marks are 74, i.e. 37%. **Ans. 37**

14. $n = 123112233111222333 \dots \overbrace{111 \dots 1}^{10 \text{ times}} \overbrace{22 \dots 2}^{10 \text{ times}} \overbrace{33 \dots 3}^{10 \text{ times}} 3$. Let K denotes the number of digit of n . Find $\frac{K}{5}$.

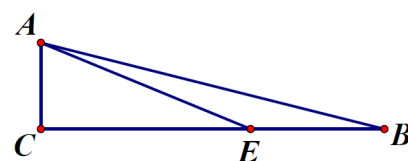
Solution: Each of 1, 2, 3 occurs $1 + 2 + 3 + \dots + 10 = 55$ times. So, this is a 165 digit number. **Ans. 33.**

15. Evaluate: $\frac{1 + \sqrt{2} + \sqrt{4} + \sqrt{8}}{1 + \sqrt{2}} + \frac{1 + \sqrt{3} + \sqrt{9} + \sqrt{27}}{1 + \sqrt{3}}$.

Solution: $\frac{1 + \sqrt{2} + \sqrt{4} + \sqrt{8}}{1 + \sqrt{2}} + \frac{1 + \sqrt{3} + \sqrt{9} + \sqrt{27}}{1 + \sqrt{3}} = \frac{1 + \sqrt{2} + 2(1 + \sqrt{2})}{1 + \sqrt{2}} + \frac{1 + \sqrt{3} + 3(1 + \sqrt{3})}{1 + \sqrt{3}} = 3 + 4 = 7$. **Ans. 7.**

16. In $\triangle ABC$, $\angle C = 90^\circ$. $AC : BC = 1 : 4$. Point E is on BC such that $BE : CE = 2 : 3$. If $AB = 35\sqrt{17}$, find AE .

Solution: Let $AC = x, BC = 4x$, so $x^2 + (4x)^2 = (35\sqrt{17})^2 \Rightarrow x = 35 \Rightarrow AC = 35, BC = 140 \Rightarrow BE = 56, CE = 84 \Rightarrow AE^2 = AC^2 + CE^2 = 35^2 + 84^2 \Rightarrow AE = 91$. **Ans. 91.**



17. If the principle is increased by Rs. 10,000 then simple interest increases by Rs. 2,100 in the period of 3 years. Find the rate of interest per cent per annum.

Solution: So, interest on 10000 is 700 per year. **Ans. 7.**

18. A alone completes a piece of work in 4 days and B alone completes it in 6 days. If A and B work on alternate days and B starts on first day, how many days are required to complete the work?

Solution: A completes $\frac{1}{4}$ work in one day. B completes $\frac{1}{6}$ work in one day. So, in two consecutive days, they complete $\frac{5}{12}$ work. So, in four days, they will complete $\frac{5}{3}$ work which leaves $\frac{1}{6}$ work, which B will complete on the fifth day. **Ans. 5.**

19. LCM of the LCM and GCD of 2 non-coprime numbers a and b is 96. What is the LCM of a and b ?

Solution: Observe that GCD of two numbers always divides LCM , so LCM of LCM and GCD is LCM only. **Ans. 96.**

20. A CFL bulb costs 100 and lasts for 2.5 years while a regular bulb costs 15 and lasts for 5 months. By what percentage should the price of CFL bulb be reduced to break even with the regular bulb.

Solution: 2.5 years is 30 months. So, we will need 6 regular bulbs during the same period, which will cost 90 rupees. So, price of the CFL bulb should be reduced by 10%. **Ans. 10**

21. Each square of the grid is filled with one of the number 1, 2, 3, 4 such that each row, column and both diagonals contain all 4 numbers. Mark the largest number formed by the digits in the first two squares (marked portion).

Solution: With a little trying, you can see that the following is the required configuration:

4	3	2	1
2	1	4	3
1	2	3	4
3	4	1	2

Ans. 43.

			1
		4	
1			
			2

22. When $3x^2 - ax + 8$ is divided by $(x - 2)$ and $5x^2 + ax - 17$ is divided by $(x + 3)$, the remainders are the same. Find a .

Solution: $3x^2 - ax + 8 = 3x(x - 2) + (6 - a)(x - 2) + 20 - 2a$ so remainder is $20 - 2a$. Similarly $5x^2 + ax - 17 = 5x(x + 3) + (a - 15)(x + 3) + 28 - 3a$. So, we get $20 - 2a = 28 - 3a \Rightarrow a = 8$.

Ans. 8.

23. Fig 1 shows 6 equilateral triangle are arranged to form a regular hexagon of side a . In Fig 2, a regular hexagon of side $2a$ is formed with 24 equilateral triangles. If a regular hexagon of length $5a$ is formed by n equilateral triangles, find $\frac{n}{6}$.

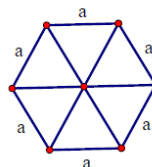


Fig 1

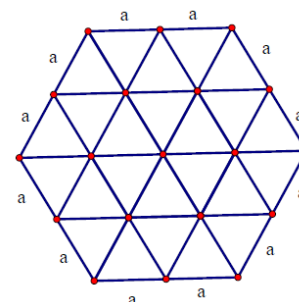


Fig 2

Solution: Observe that the pattern of number of triangles is $6(1), 6(4), 6(9), 6(16), 6(25)$ So, $n = 6(25)$. **Ans. 25.**

24. Find the value of $\frac{\sqrt{28.8} + \sqrt{72} + \sqrt{43.2}}{\sqrt{0.2} + \sqrt{0.3} + \sqrt{0.5}}$.

Solution: $\frac{\sqrt{28.8} + \sqrt{72} + \sqrt{43.2}}{\sqrt{0.2} + \sqrt{0.3} + \sqrt{0.5}} = \frac{12(\sqrt{0.2} + \sqrt{0.5} + \sqrt{0.3})}{\sqrt{0.2} + \sqrt{0.3} + \sqrt{0.5}} = 12$ **Ans. 12**

Answers

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	2	32	2	32	95	30	30	5	27	8	16	1
Q.No.	13	14	15	16	17	18	19	20	21	22	23	24
Ans.	37	33	7	91	7	5	96	10	43	8	25	12

Q 1. If $n = (2 \times 10^{100}) + 3$, then find the sum of the digits of n^2 .

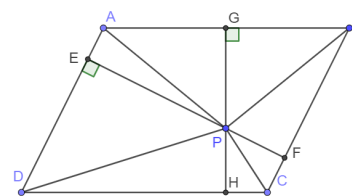
Solution: $n^2 = (2 \times 10^{100})^2 + 2(2 \times 10^{100})(3) + 3^2 = 4 \times 10^{200} + 12 \times 10^{100} + 9$ So, the digits appearing are 4, 1, 2, 9. **Ans. 16**

Q 2. n is a natural number divisible by 75. If n^2 is written in decimal notation as $5k05h25$, then find the value of $k^2 + 4h$.

Solution: Observe that n^2 is divisible by 625 (and 9). Note that $5k05h25 = 5000000 + k(100000) + 5000 + 100h + 25$ and 5000 is divisible by 625, so 100000 and 5000000 are also divisible by 625. So, $100h + 25$ must be divisible by 625, so we get $h = 6$. Since the number is divisible by 9, sum of the digits must be divisible by 9. So, we get $k = 4$. **Ans. 40.**

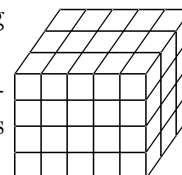
Q 3. $ABCD$ is a parallelogram and P is a point in its interior. If the areas of $\triangle APB$, $\triangle BPC$ and $\triangle CPD$ are 18, 25 and 39 respectively, then find the area of $\triangle DPA$.

Solution: $\text{Area}(\square ABCD) = AB \times HG = AD \times EF$. Also, $\text{Area}(\triangle PAB) = \frac{1}{2}(AB)(PG)$, $\text{area}(\triangle PCD) = \frac{1}{2}(PH)(CD)$ so, $\text{Area}(\triangle PAB) + \text{area}(\triangle PCD) = \frac{1}{2}(AB)(GH) = \frac{1}{2} \text{area}(\square ABCD)$. Similarly, we can prove that $\text{area}(\triangle PAD) + \text{area}(\triangle PBC) = \text{area}(\square ABCD)$ which gives $\text{area}(\triangle PAB) + \text{area}(\triangle PCD) = \text{area}(\triangle PAD) + \text{area}(\triangle PBC) \Rightarrow 18 + 39 = 25 + \text{area}(\triangle APD) \Rightarrow \text{area}(\triangle APD) = 32$. **Ans. 32.**



Q 4. Consider a wooden cuboid of size $3 \times 4 \times 5$. If a cubical block of size $1 \times 1 \times 1$ is cut away from each of its corners, find the total surface area of the resulting object.

Solution: When we cut a corner, we remove three faces of it, but the corresponding three faces of the inner blocks become exposed, so area effectively does not change. So, $\text{area} = 2(3 \times 4 + 4 \times 5 + 5 \times 3) = 94$. **Ans. 94.**



Q 5. Eleven batsmen score runs as follows:

- The first five players together score 4 times the total score of the remaining players;
- The first six players together score 5 times the total score of the remaining players; and
- The average score of all eleven players is 30.

Find the score of the sixth batsman.

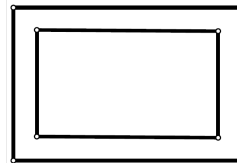
Solution: Since average is 30, total runs = 330. First condition tells us that total runs scored by first five players is 264. Second condition tells us that total runs scored by first six players is 275. So, runs scored by sixth player = $275 - 264 = 11$. **Ans. 11.**

Q 6. From 3:00 AM to 3:00 PM, determine the number of times that the hour hand and minute hand are symmetrically placed with respect to line corresponding to six o'clock position.

Solution: For the first time after 3, approximately at 3.40, the hands will be symmetrical. Then once in every hour, i.e. between 4 and 5, 5 and 6 etc. Additionally, at 6 o'clock, the hands are symmetric. So, totally thirteen times. **Ans. 13.**

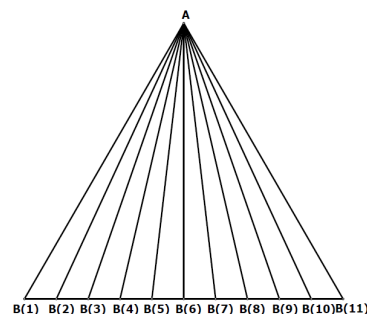
Q 7. As shown in the figure, a strip with a constant width of 1 meter is paved along the boundary inside a rectangular ground, whose length and breadth are in the ratio 3 : 2. If the area of the paved region is 46 square meters, then find the outer perimeter of the ground in meters.

Solution: Let the length and breadth be $3x, 2x$. So, inner rectangle is $(3x - 2) \times (2x - 2)$. So, we get $(3x)(2x) - (3x - 2)(2x - 2) = 46 \Rightarrow x = 5$. So, outer dimensions are 15×10 , so perimeter = $2(15 + 10) = 50$ **Ans. 50.**



Q 8. As shown in the figure, $B_1, B_2, \dots, B_{11}$ are eleven equidistant points lying on a straight line, in that order. Point A is chosen so that AB_1B_{11} is an equilateral triangle. A is joined to each of B_1 to B_{11} . Find the total number of acute-angled triangles in the given figure.

Solution: To form acute angled triangle, we have to choose any one of B_1 to B_5 and any one of B_7 to B_{11} and A . **Ans. 25.**



Q 9. A square array of numbers is said to be a magic square, if the sum of the numbers in each of its row, column and main diagonals is equal. In the following 4×4 magic square, find $(a + b + c + d)$ if the sum of the remaining 12 entries is 150.

Solution: Since the sum of each row, column and diagonal is same, its equal to $a + b + c + d$. So, total sum of all entries is four times $a + b + c + d$. So, we get $4(a + b + c + d) = 150 + (a + b + c + d) \Rightarrow a + b + c + d = 50$. **Ans. 50.**

a			
	b		
		c	
			d

Q 10. Find two digit natural number n such that the greatest common divisor of 2520 and $n + 100$ is 21.

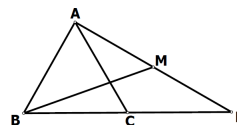
Solution: $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$. Since the GCD is 21, the number is divisible by 21 and not divisible by 2, 5 and 9. Since n is a two digit number, $n + 100$ is between 110 and 199. The only such number is 147. **Ans. 47.**

Q 11. Hardik takes 50 minutes to reach his school from his home, when he walks at a constant speed. If he wants to reach his school in 40 minutes, then by what percent should he increase his walking speed?

Solution: Suppose his current speed is v m/min and distance to school is d meters. So, we have $\frac{d}{v} = 50 \Rightarrow v = \frac{d}{50}$. Suppose new speed is v_1 , so we need $\frac{d}{v_1} = 40 \Rightarrow v_1 = \frac{d}{40}$. So, we have $\frac{v_1}{v} = \frac{5}{4}$, which means he needs to increase his speed by 25%. **Ans. 25**

Q 12. $\triangle ABC$ is an equilateral triangle. D is a point on line BC such that C is the midpoint of BD . Let M be the midpoint of AD . If $AB = 4$, then find BM^2 .

Solution: Observe that $AC = CD \Rightarrow \angle CAD \cong \angle CDA$, also $m\angle CAD + m\angle CDA = 60 \Rightarrow m\angle CAD = m\angle CDA = 30$, so $\triangle ABD$ is a $30 - 60 - 90$ triangle. So, $AB = 4 \Rightarrow AD = 4\sqrt{3} \Rightarrow AM = 2\sqrt{3} \Rightarrow BM^2 = AB^2 + AM^2 = 16 + 12 = 28$. **Ans. 28.**



Q 13. Given a rhombus whose area is 240, and one of the diagonals is 16; find the perimeter of the rhombus.

Solution: Consider a rhombus with diagonals d_1, d_2 . Then the area of the rhombus is $\frac{d_1 d_2}{2}$. So, here, $240 = \frac{16 d_2}{2} \Rightarrow d_2 = 30$. Also from Pythagoras theorem, we have the side of the

$$\text{rhombus} = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{8^2 + 15^2} = 17. \text{ Ans. 68.}$$

Q 14. Let $a * b$ denote the length of the hypotenuse of a right-angled triangle whose remaining two sides have lengths a, b . If $(12 * 16) * k = 25$, then find the value of k .

Solution: $12 * 16 = \sqrt{12^2 + 16^2} = 20$. So, $(12 * 16) * k = 25 \Rightarrow \sqrt{20^2 + k^2} = 25 \Rightarrow k = 15$.

Ans. 15.

Q 15. n is a natural number such that the least common multiple of n and 72 is 360. Find the average of all possible values of n between 52 and 98, which satisfy the above condition.

Solution: $72 = 2 \times 2 \times 2 \times 3 \times 3$ and $360 = 5 \times 72$, so n must have a factor of 5 appearing only once, whereas 2 can appear in the factor either once, twice or thrice (or zero times) and 3 can appear either once or twice (or zero times). The numbers having this factorisation between 52 and 98 are 60 and 90. **Ans. 75.**

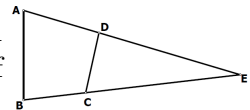
Q 16. Find the value of $\left[\frac{4}{\sqrt{7}-\sqrt{3}} + \frac{2}{\sqrt{5}+\sqrt{3}}\right]^2 - \sqrt{140}$.

Solution: $\left[\frac{4}{\sqrt{7}-\sqrt{3}} + \frac{2}{\sqrt{5}+\sqrt{3}}\right]^2 - \sqrt{140} = \left[\frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} + \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}\right]^2 - \sqrt{140}$
 $= [\sqrt{7} + \sqrt{5}]^2 - \sqrt{140} = 12 + 2\sqrt{35} - \sqrt{140} = 12. \text{ Ans. 12.}$

Q 17. $ABCD$ is a cyclic quadrilateral. Lines AD and BC meet at point E .

If $m\angle BAD = 75^\circ$ and $\angle ADC = 100^\circ$, then find $m\angle AEB$.

Solution: We use the property that opposite angles of a cyclic quadrilateral are supplementary. This gives $m\angle ABC = 80^\circ$. So, in $\triangle ABE$ (sum of angles in a triangle is 180), we get $m\angle AEB = 180 - (75 + 80) = 25$. **Ans. 25.**



Q 18. n is a two-digit number whose ten's digit is a , and unit's digit is b . When we interchange the digits, the value of the number increases by 20%. Find the value of n .

Solution: We get $10b + a = 1.2(10a + b) \Rightarrow 50b + 5a = 60a + 6b \Rightarrow 44b = 55a \Rightarrow 4b = 5a$. Since a and b are single digit numbers, this is possible only when $b = 5, a = 4$. **Ans. 45.**

Q 19. An ant is at point A of a segment AB , having length 12. The ant moves as follows: First it moves 3 units towards B , then moves 2 units towards A ; again moves 3 units towards B , then moves 2 units towards A , and so on. Find the total distance traveled by the ant, when it reaches point B for the first time.

Solution: Effectively, the ant moves 1 unit ahead when it has traveled 3 forward and 2 backward that is totally five units. This way, when it moves 9 units forward, it would have traveled 45 units distance. Now, it moves forward by 3 units and it has reached point B . **Ans 48.**

Q 20. In a party, each boy dances with exactly three girls, and each girl dances with exactly four boys. If a total of 35 students are present at the party, how many boys are there?

Solution: Suppose there are four boys and three girls, then the above condition is satisfied. This requires a total of 7 students. Since there are totally 35 students, we will need five such groups. So, there are 5×4 boys. **Ans. 20.**

Q 21. If m, n are distinct natural numbers, then find the value of

$$\frac{1}{1+2025^{m-n}} + \frac{1}{1+2025^{n-m}} + \sqrt{2025}.$$

Solution: Note: $\sqrt{2025} = 45$. Observe that $2025^{n-m} = \frac{1}{2025^{m-n}}$, so we get

$$\frac{1}{1+2025^{m-n}} + \frac{1}{1+2025^{n-m}} + \sqrt{2025} = \frac{1}{1+2025^{m-n}} + \frac{1}{1+\frac{1}{2025^{m-n}}} + 45 = \frac{1}{1+2025^{m-n}} + \frac{2025^{m-n}}{2025^{m-n}+1} + 45 = \frac{1+2025^{m-n}}{2025^{m-n}+1} + 45 = 1 + 45 = 46. \text{ Ans. 46.}$$

Q 22. A bag contains several 50-paise, 1-rupee and 2-rupee coins, in the ratio 8:6:7 respectively. If the value of the contribution of 2-rupee coins is more by 12 rupees than the contribution of all the remaining coins, then find the total number of coins.

Solution: Suppose there $8x$ 50 paise coins, $6x$ 1 rupee coins and $7x$ 2 rupee coins. So, contribution of 2 rupee coins is $14x$ rupees. Remaining contribution is $(0.5)(8x) + 6x = 10x$ rupees.

So, $14x - 10x = 12 \Rightarrow x = 3$. So, total coins are $21x = 63$. **Ans. 63.**

Q 23. In an examination, the ratio of the number of students who passed to the number of students who failed is 4:1. If 20 more students had appeared, and 2 more had passed, then the ratio would have been 2:1. Find the number of students who appeared in the exam originally.

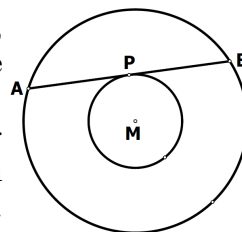
Solution: Suppose there are $4x$ passed and x failed students. When 2 more students passed, total students passed is $4x + 2$. Total 20 new students appeared, means 18 more failed. So, new failed students is $x + 18$. So, we have $\frac{4x+2}{x+18} = 2 \Rightarrow 4x + 2 = 2x + 36 \Rightarrow x = 17$, so initially $4x + x = 5x$ students appeared, i.e. $5(17) = 85$. **Ans. 85.**

Q 24. Person A can do a piece of work in 26 days. Person B is 30% more efficient than A . In how many days can B complete the same work?

Solution: A can do $\frac{1}{26}$ work in a day. So, B completes $(\frac{1}{26}) 1.3 = \frac{1}{20}$ work in a day. So, B needs 20 days to complete the work. **Ans. 20.**

Q 25. As shown in the diagram, two concentric circles have a common center M . P is a point on the smaller circle. A line is constructed perpendicular to MP at P , which meets the bigger circle in points A, B . Given $AB = 12$, and if the area of the region/ring between the two circles is $k\pi$, then find the value of k .

Solution: Since $AP = PB$ we have $AP = 6$. Suppose the radius of bigger circle is r_2 and that of smaller circle is r_1 . Using Pythagoras theorem in $\triangle AMP$, we get $MA^2 = MP^2 + PA^2 \Rightarrow r_1^2 = r_2^2 + 36 \Rightarrow \pi(r_1^2 - r_2^2) = 36\pi$. So, $k = 36$. **Ans. 36.**



Answers

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	
Ans.	16	40	32	94	11	13	50	25	50	47	25	28	
Q.No.	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	68	15	75	12	25	45	48	20	46	63	85	20	36

Q1. Virat runs twice as fast as he walks. He travels from his house to school by walking some distance and by running some distance. On Monday his walking time is twice his running time and reaches the school in 30 minutes. On Tuesday his running time is twice his walking time. Find the time in minutes he takes to reach the school on Tuesday.

Solution: Virat's running speed is twice his walking speed. On Monday Virat walks for 20 minutes and runs for 10 minutes. Hence on Monday distance covered by walking equals distance covered by running. This means that, if Virat had covered all the distance walking only he would take 40 minutes. (1) On Tuesday his running time is twice his walking time. Hence distance covered by running is 4 times distance covered by walking. Hence distance covered walking is $\frac{1}{5}$ the total distance. By (1) Virat must have walked $40 \times \frac{1}{5} = 8$ minutes on Tuesday. Hence he must have run for 16 minutes on Tuesday. So total time taken on Tuesday is 24 minutes. **Ans. 24.**

Q2. The fraction $\frac{35}{16}$ can be written in the form $\frac{35}{16} = 2 + \frac{1}{x + \frac{1}{y}}$ where x, y are natural numbers.

Find $(x + y)^2$.

Solution: $\frac{35}{16} = 2 + \frac{3}{16} = 2 + \frac{1}{\frac{16}{3}} = 2 + \frac{1}{5 + \frac{1}{3}}$ So, $x = 5, y = 3, (x + y)^2 = 64$. **Ans. 64.**

Q3. Let $n = 100^{25} - 25$. Let S denote the sum of the digits of n . Find the smallest natural number k such that $S + k$ is a perfect square.

Solution: 100^{25} means 1 followed by 50 zeroes.

$\therefore n = 9999 \dots 975$, where digit 9 occurs 48 times. Hence the sum of the digits of n namely $S = (48 \times 9) + 7 + 5 = 444$. The nearest square is 484. So, answer is 40. **Ans. 40.**

Q4. If $1.236 \times 10^{15} - 5.23 \times 10^{14} = a.bc \times 10^k$ where a, b, c are digits from 1 to 9 and k is a natural number. Find $(a + b + c + k)$.

Solution: $1.236 \times 10^{15} - 5.23 \times 10^{14} = 12.36 \times 10^{14} - 5.23 \times 10^{14} = (12.36 - 5.23) \times 10^{14} = 7.13 \times 10^{14}$. So, answer is $7 + 1 + 3 + 14 = 25$. **Ans. 25.**

Q5. Two adults have their birthday on the same day. On a particular birthday the product of their ages is 770. Find the sum of their ages on that birthday.

Solution: As each person is an adult, we need to find two factors greater than or equal to 18. $770 = 11 \times 7 \times 5 \times 2 = 35 \times 22$. So, answer is $35 + 22 = 57$. **Ans. 57.**

Q6. Mohit bought a number of balls. He was required to pay 5% tax on his purchase. If he did not have to pay the tax he could have bought 3 more balls in the total amount he had spent. How many balls did Mohit buy?

Solution: One can assume that cost price of 1 ball is Rs. 100. Tax is 5%. Hence he pays Rs. 5 as tax per ball. As he could buy 3 balls from tax paid means he has paid $100 \times 3 = 300$ as tax. Hence he has bought $\frac{300}{5} = 60$ balls. **Ans. 60.**

Q7. Two candles of length one foot each start burning at the same time. One of the candles will burn down in 40 hours and the other in 24 hours. If after H hours one of the candle has length 3 times the length of the other candle, find H .

Solution: The candle which completely burns in 40 hrs. (say candle A) will burn $\frac{1}{40}$ feet in one hour.

Therefore in H hours candle A will burn $\frac{H}{40}$ feet. So, after H hours, $(1 - \frac{H}{40})$ feet of candle A remains.

Similarly,

The candle which completely burns in 24 hours (say candle B), will burn $\frac{1}{24}$ feet in one hour. Therefore in H hours, candle B will burn $\frac{H}{24}$ feet. So, after H hours, $(1 - \frac{H}{24})$ feet of candle B

remains.

As candle A burns slower than candle B , after H hours, candle A will be longer than candle B , Therefore $1 - \frac{H}{40} = 3 \left(1 - \frac{H}{24}\right)$,

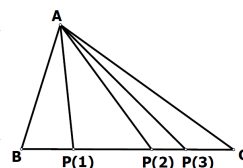
Solving, we get $H = 20$. **Ans. 20.**

Q8. 125 small cubes of size $1 \times 1 \times 1$ are put together to form a cube of size $5 \times 5 \times 5$. Two cubes of size $1 \times 1 \times 1$ are said to be neighbours if they are placed such that their one face of size 1×1 touches each other. Find the number of $1 \times 1 \times 1$ cubes having exactly four neighbours.

(Note that a cube has 8 vertices, 12 edges and 6 faces.)

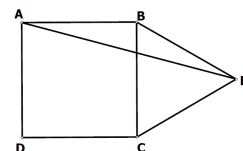
Solution: Observe that cubes having four neighbours are the ones along every edge, excluding the ones at the corner. So every edge has three such cubes. Total edges = 12, hence answer is 36. **Ans. 36.**

Q9. In $\triangle ABC$, three points P_1, P_2, P_3 are placed on segment BC and each joined to vertex A . The resulting figure contains all together 10 triangles. Find the total number of triangles present in the figure if 12 points $P_1, P_2, \dots, P_{11}, P_{12}$ are placed on segment BC and each is joined to vertex A .



Solution: Since vertex A is common to all triangles, let's write only the bases of the triangles. In the first case, let's see how we can methodically write all the bases. Each base has left end point and right end point. If the left point is B then the possible bases are BP_1, BP_2, BP_3, BC . So, there are four bases. If we take P_1 as the left point then possible bases are P_1P_2, P_1P_3, P_1C , i.e. there are three bases. With P_2 as left point we get P_2P_3, P_2C as two bases and with P_3 there is one base P_3C . Therefore there are $4 + 3 + 2 + 1 = 10$ possible bases hence 10 triangles. In the given problem, with B as left point we have bases $BP_1, BP_2, BP_3, \dots, BP_{12}, BC$ giving 13 bases hence total number of triangles is $13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 91$. **Ans. 91.**

Q10. As shown in the figure $\square ABCD$ is a square and $\triangle EBC$ is an equilateral triangle. Find the measure of $\angle DAE$.



Solution: $m\angle ABE = 90^\circ + 60^\circ = 150^\circ$. In $\triangle ABE$, note that $AB = BE$. So, $m\angle BAE = \frac{1}{2}(180 - 150) = 15^\circ$. So, $m\angle DAE = 90^\circ - 15^\circ = 75^\circ$ **Ans. 75.**

Q11. A polygon $A_1A_2A_3\dots A_{10}$ of 10 sides is inscribed in a circle, that is all the vertices lie on the circumference of the same circle. All the 10 sides are of equal length. Let M be the center of the circle.

Find the measure of $\angle A_3A_1A_2$.

Solution:

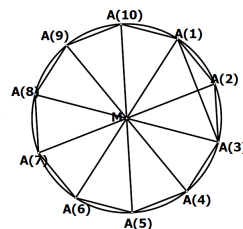
Note that each angle $\angle A_1MA_2, \angle A_2MA_3, \angle A_3MA_4, \dots, \angle A_{10}MA_1$, has measure 36° .

In $\triangle A_1MA_2$, $MA_1 = MA_2$.

$\therefore m\angle MA_1A_2 = m\angle MA_2A_1$.

$\therefore m\angle MA_1A_2 = m\angle MA_2A_1 = \frac{1}{2}(180 - 36) = 72^\circ$.

Similarly in $\triangle MA_2A_3$, $m\angle MA_2A_3 = 72^\circ \therefore m\angle A_1A_2A_3 = 144^\circ$.



In $\triangle A_1A_2A_3$, $A_1A_2 = A_2A_3 \therefore m\angle A_3A_1A_2 = \frac{1}{2}(180 - 144) = 18^\circ$. **Ans. 18.**

Q12. In $\triangle ABC$ D is on segment BC such that $BD = x$ and $DC = 2x$. M is the midpoint of segment AC . If area of $\triangle ABD$ is 26 units find the area of $\triangle BMA$.

Solution: Let us say that the length of altitude from A on BC is h .

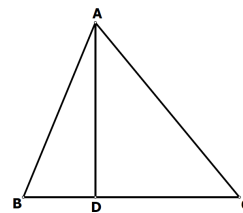
$\therefore \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle ABD)} = \frac{\frac{1}{2}BC \times h}{\frac{1}{2}BD \times h} = \frac{BC}{BD} = 3 \therefore \text{Area}(\triangle ABC) = 78$.

Exactly by the same logic, $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle BMA)} = 2 \therefore \text{Area}(\triangle BMA) = 39$. **Ans. 39.**

Q13. In $\triangle ABC$, D is the foot of the altitude from A on segment BC . If $AD = 24$, $AC = 30$ and $BC = 28$. Find AB .

Solution: In $\triangle ADC$, by Pythagoras theorem
 $DC^2 = AC^2 - AD^2 = 30^2 - 24^2 = 900 - 576 = 324 = 18^2$
 $\therefore BD = BC - DC = 10$

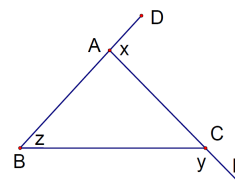
In $\triangle ADB$, we get by Pythagoras theorem
 $AB^2 = AD^2 + DB^2 = 24^2 + 10^2 = 676 = 26^2 \quad \therefore AB = 26$. **Ans. 26.**



Q14. As shown in the figure $m\angle DAC = x^\circ$, $m\angle BCE = y^\circ$ and $m\angle ABC = z^\circ$. Find $\frac{1}{12}(x + y - z)$.

Solution: Solution: Let A, B, C denote the measures of angles of $\triangle ABC$.
 Note that $A = 180 - x$ and $C = 180 - y$. We have

$$\begin{aligned} A + B + C &= 180 \\ \therefore (180 - x) + z + (180 - y) &= 180 \\ \therefore x + y - z &= 180 \\ \therefore \frac{1}{12}(x + y - z) &= 15 \end{aligned}$$



Ans. 15.

Q15. I have a 14 digit interesting number. The sum of its any three consecutive digits is same. If its first digit is 4, its 5th digit is 7 and the sum of its all digits is 79 then find the sum of its last 4 digits.

Solution: Let the first three digits be x, y, z . As the sum of three consecutive digits is same, the fourth digit must be x . It follows that the fifth digit must be y , sixth must be z and so on. Therefore the number must be $xyzxyzxyzxyz$. $\therefore x = 4, y = 7$. Sum of all digits $= 5x + 5y + 4z = 79$. This gives $z = 6$. So sum of last four digits $= y + z + x + y = 7 + 4 + 6 + 7 = 24$. **Ans. 24.**

Q16. Let $n = (7584)^2 + 4(7584)(1208) + 4(1208)^2$. Find the smallest value of the natural number m such that the product of m and n is a perfect cube.

Solution: Solution: Let $x = 7584$ and $y = 1208$. Hence n can be written as; $n = x^2 + 4xy + 4y^2 = (x + 2y)^2 = (7584 + 2(1208))^2 \therefore n = (10000)^2 = 2^8 5^8$. So, $m = 2 \times 5 = 10$. **Ans. 10.**

Q17. Let m be the largest divisor of 72^3 other than itself. Let n be the largest divisor of 75^4 other than itself. If L.C.M. of m and $n = p^a q^b r^c$ where p, q, r are distinct prime numbers then find the value of $a + b + c$.

(Note that the largest divisor of 10^3 other than 10^3 is 500).

Solution: $72 = 2^3 3^2 \therefore 72^3 = (2^3 3^2)^3 = 2^9 3^6 \therefore m = 2^8 3^6$.

Similarly, $75 = 5^2 3 \therefore 75^4 = (5^2 3)^4 = 5^8 3^4 \therefore n = 5^8 3^3$.

So, LCM of m and n is $2^8 3^6 5^8$. So, $a + b + c = 8 + 6 + 8 = 22$. **Ans. 22.**

Q18. We define a new operation $*$ as given below.

$a * b = a^2 + b$. For example, $8 * 5 = 8^2 + 5 = 69$.

If n is a natural number and p is a prime number then find the smallest value of p satisfying $8 * p = n * 6$.

Solution: We want $8 * p = n * 6 \therefore 64 + p = n^2 + 6 \therefore p = n^2 - 58$. So, n^2 must be greater than 58 and $n^2 - 58$ must be prime. Try $n = 8$, gives $n^2 - 58 = 8$ is composite. Try $n = 9$ gives $n^2 - 58 = 23$ which is a prime. **Ans. 23.**

Q19. Find the value of $(2\sqrt{20} + 4\sqrt{8})(6\sqrt{2} - \sqrt{45})$.

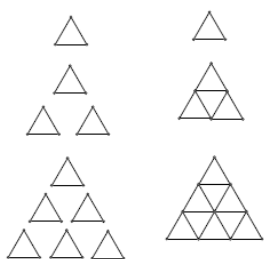
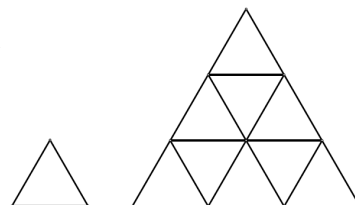
Solution: Solution: $\sqrt{20} = 2\sqrt{5}, \sqrt{8} = 2\sqrt{2}, \sqrt{45} = 3\sqrt{5}$
 $\therefore (2\sqrt{20} + 4\sqrt{8})(6\sqrt{2} - \sqrt{45}) = (4\sqrt{5} + 8\sqrt{2})(6\sqrt{2} - 3\sqrt{5})$
 $= 12(\sqrt{5} + 2\sqrt{2})(2\sqrt{2} - \sqrt{5})$

$$\begin{aligned}
&= 12(2\sqrt{2} + \sqrt{5})(2\sqrt{2} - \sqrt{5}) \\
&= 12((2\sqrt{2})^2 - (\sqrt{5})^2) \\
&= 12(8 - 5) = 36. \quad \text{Ans. 36.}
\end{aligned}$$

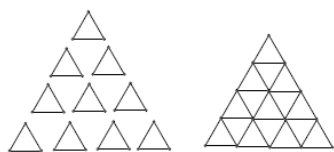
Q20. If $a^2 - b^2 = 132$ and $a + b = 22$ then find the value of $a + b^2$.

Solution: $a^2 - b^2 = (a + b)(a - b)$ substituting given value we get, $a - b = 6 \dots (1)$.
 $a + b = 22 \dots (2)$. Adding equation (1) and (2) we get $2a = 28$. This gives $a = 14$ and $b = 8$. $\therefore a + b^2 = 14 + 64 = 78$. **Ans. 78.**

Q21. As shown in the following figure equilateral triangle of size 1 is formed using 3 match sticks. To form an equilateral triangle of size 3 with all the equilateral triangle of size 1 inside it we require 18 match sticks. Find the total number of match sticks required to construct a size 6 equilateral triangle with all the equilateral triangle of size 1 inside it.



Solution: Observe the pattern: So, the answer must be $3(1 + 2 + 3 + 4 + 5 + 6) = 63$. **Ans. 63.**



Q22. Consider the 8-digit number $3681m42n$ where m and n are digits from 0 to 9. Find the value of $m^2 + n$ if given 8-digit number is divisible by 72.

Solution: Solution: 72 is divisible by 8 and 9. So, the 8-digit number must be divisible by 8 and 9. So, $42n$ is divisible by 8 giving $n = 4$. Now, the sum of the digits must be divisible by 9 giving $m = 8$. So, $m^2 + n = 64 + 4 = 68$. **Ans. 68.**

Q23. Find the value of $100(1 - \frac{1}{8})(1 - \frac{1}{9})(1 - \frac{1}{10}) \dots (1 - \frac{1}{20})$.

Solution: Observe: $1 - \frac{1}{8} = \frac{7}{8}, 1 - \frac{1}{9} = \frac{8}{9}, \dots, 1 - \frac{1}{20} = \frac{19}{20}$

$$\begin{aligned}
&\therefore 100 \left(1 - \frac{1}{8}\right) \left(1 - \frac{1}{9}\right) \left(1 - \frac{1}{10}\right) \dots \left(1 - \frac{1}{20}\right) \\
&= 100 \left(\frac{7}{8}\right) \left(\frac{8}{9}\right) \left(\frac{9}{10}\right) \dots \left(\frac{18}{19}\right) \left(\frac{19}{20}\right) = 100 \left(\frac{7}{20}\right) = 35.
\end{aligned}$$

Ans. 35.

Q24. Integer k is 4^{th} power of another integer. If 18 is a factor of k then find the smallest value of $\frac{k}{18}$.

Solution: $18 = 2^1 \times 3^2$. So, the additional minimum factors in k to make it perfect 4^{th} power are $2^3 \times 3^2$. So, the smallest $k = 2^4 \times 3^4$ giving $k/18 = 2^3 \times 3^2 = 72$ **Ans. 72.**

Q25. Club G has several members. Average age of members of G increases by one year if either five members each 9 year old leave G or new five members each 17 year old join G . Find the present number of members in G .

Solution: Suppose the present number of members is n and the average age is y years. So, the present sum of the ages is ny .

When five members each 9 years old leave, the number of members becomes $n - 5$ and sum of

ages becomes $ny - 45$ giving $ny - 45 = (y + 1)(n - 5)$.

$$\therefore ny - 45 = ny + n - 5y - 5$$

$$\therefore 5y - n = 40 \cdots (1)$$

When five members each 17 years old join the group,

number of members becomes $n + 5$ and sum of ages becomes $ny + 85$

giving $ny + 85 = (n + 5)(y + 1)$

$$\therefore ny + 85 = ny + 5y + n + 5$$

$$\therefore 5y + n = 80 \cdots (2)$$

Adding equation (1) and (2) we get $10y = 120$ giving $y = 12$ and $n = 20$ Hence the answer is 20. **Ans. 20.**

Answers:

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	
Ans.	24	64	40	25	57	60	20	36	91	75	18	39	
Q.No.	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	26	15	24	10	22	23	36	78	63	68	35	72	20